Checking Two Structural Properties of Vector Addition Systems with States

Florent Avellaneda & Rémi Morin

Université d'Aix Marseille

6 décembre 2012

VASS

Definition

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- Q *a finite set of nodes.*
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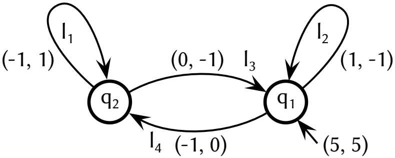

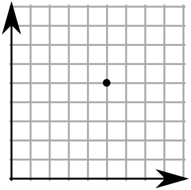

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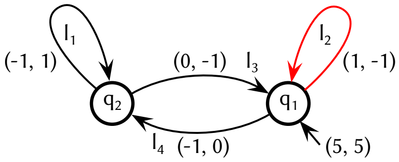

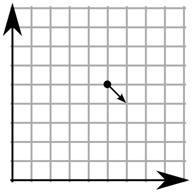

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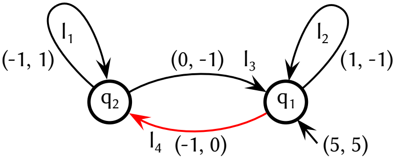

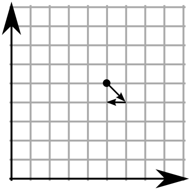

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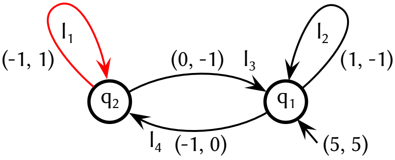

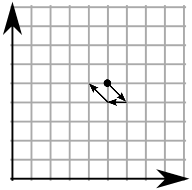

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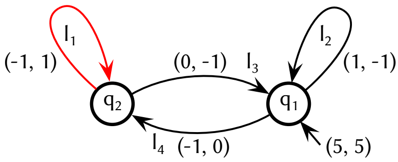

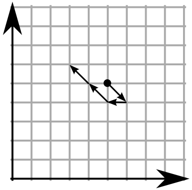

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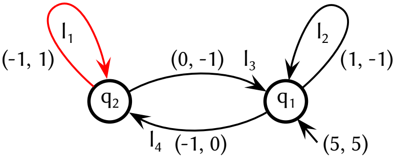

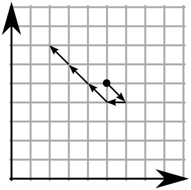

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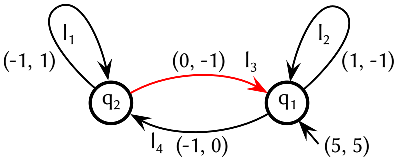

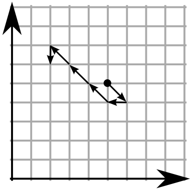

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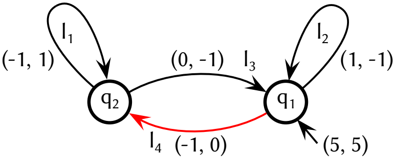

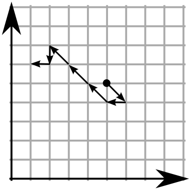

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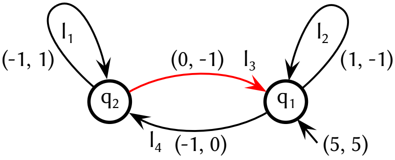

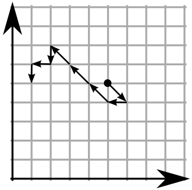

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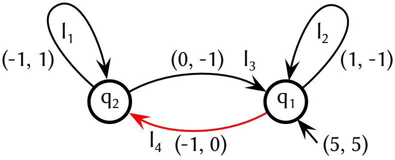
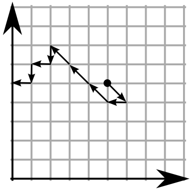
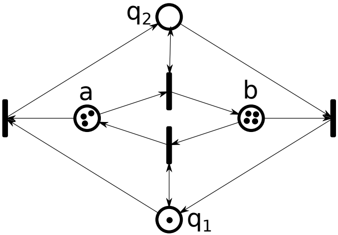


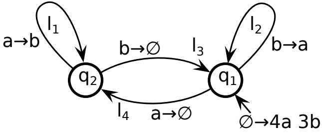


Petri net

VASS are very close to Petri nets :

- A VASS can be simulated by a Petri net.





Petri net

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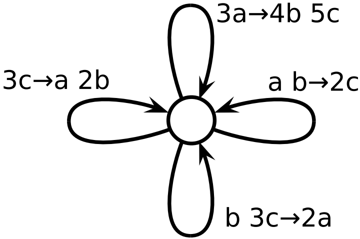
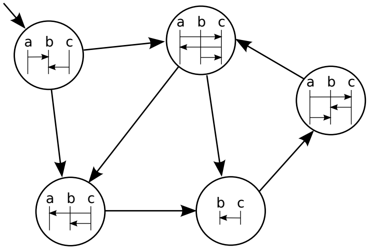
- A VASS can be simulated by a Petri net.
- A Petri net is a VASS with one state.



MSG

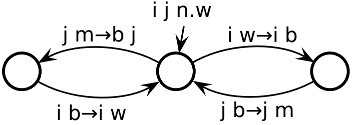
MSG (Message Sequence Graph)

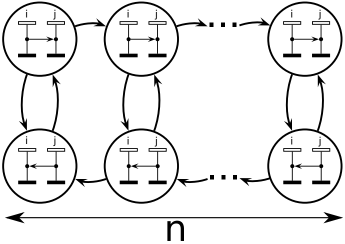
Automaton where each node is labeled by an MSC.



- MSGs are a special case of VASSs.
- VASSs are exponentially more concise than MSGs.

Example : simplified sliding window protocol





Termination

Carstensen 87

Termination of a given VASS is EXPSPACE-hard.

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But, structural termination is polynomial problem.

Property

A VASS is structurally terminating if and only if there exists no closed path whose cost is $\geq \vec{0}$.

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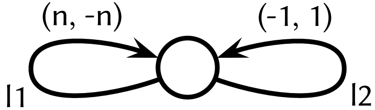
Problem solvable in polynomial time by linear programming.

Minimum length of a closed path solution is potentially exponential.



$$l_1 \underbrace{\ldots l_2 l_2 l_2 l_2 \ldots}_{n \text{ times}} \Rightarrow l_1 + n \cdot l_2$$

- The counter example is a multiset of edges.

Challenge

The counter example as multiset of edge is too complex.

Goal

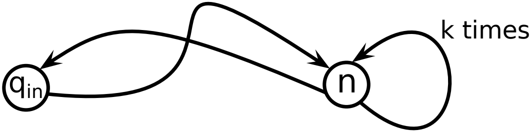
Find a simple representation of counter examples.

Solution

Use a set of lassos.

Definition

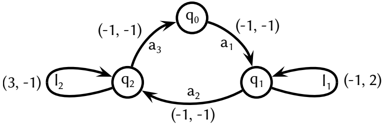
Let Y_0 be a circuit starting from node n. Let Y_1 be a simple path from q_{in} to n and Y_2 be a simple path from n to q_{in}. The closed path $Y_1 \cdot (Y_0)^k \cdot Y_2$ is called a ***lasso with valuation*** *k starting from q_{in}.*



Result

Theorem

Let H be a closed path given as a multiset of edges. We can compute in polynomial time a finite multiset of lassos S starting from $b \in V_H$ such that $cost(S) = m \times cost(H)$ with $m \in N^*$.



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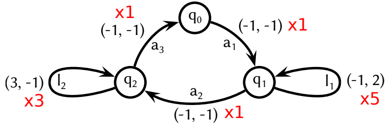
$H = a_1 + 5l_1 + a_2 + 3l_2 + a_3$



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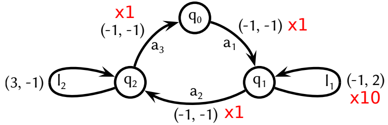

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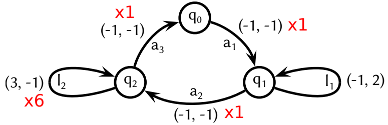

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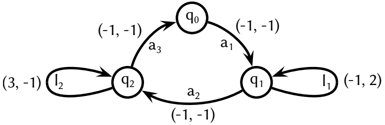

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$S = S_1 + S_2$

$cost(S) = 2 \times cost(H)$

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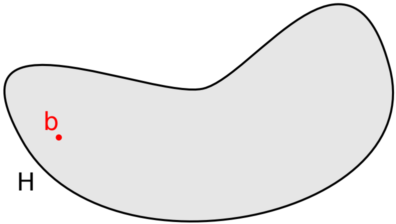

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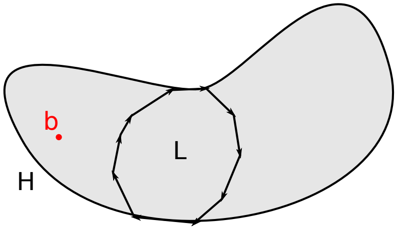

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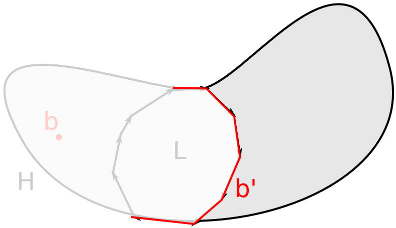

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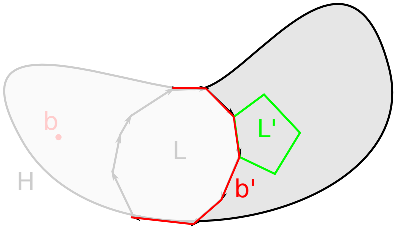

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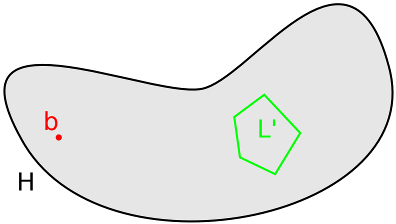

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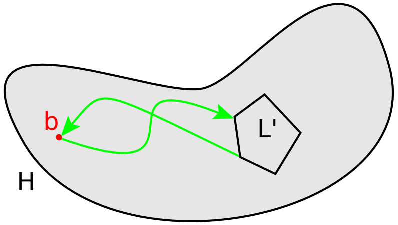

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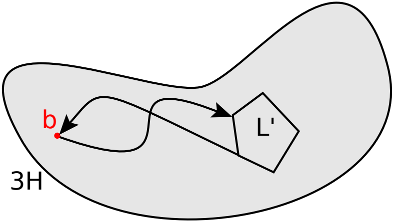

Corollary

Let H be a closed path with $b \in V_H$. Then there exist a closed path H' with almost d lassos starting from b such that $cost(H) = m \times cost(H')$.

Idea :

$x_1 \cdot cost(S_1) + x_2 \cdot cost(S_2) + ... + x_{|A|} \cdot cost(S_{|A|}) \geq \vec{0}$

By linear programming, we can find d lassos such that

$x_{i1} \cdot cost(S_{i1}) + x_{i2} \cdot cost(S_{i2}) + ... + x_{id} \cdot cost(S_{id}) \geq \vec{0}$

Conclusion

- Structural termination of VASS is equivalent to search a positive closed path.
- Linear programming solve this problem in polynomial time, but with complex counter example.
- Polynomial algorithm to represent a counter example by d lassos.

Thanks.