Verification of Concurrent Systems

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- Static/Dynamic number of threads
- Communication

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    - Channels (queues)
    - Unordered/FIFO ...
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- We assume finite data domain (e.g., booleans).

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- Product grows exponentially in # threads and # variables.
- Reachability is decidable, and PSPACE-complete.
  [Kozen, FOCS'77]

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Facing the state-space explosion

- Partial order techniques
  - Independent actions $\Rightarrow$ commutable actions $\Rightarrow$ many interleavings
  - Explore representatives up to independent actions commutations
  - Compact representations of sets of behaviors (Unfoldings)

  Godefroid, Wolper, Peled, Holzman, Valmari, McMillan, Esparza, ...

- Symbolic techniques
  - Compact representations of sets of states (e.g., BDD)
  - Encoding bounded-length computation + SAT solvers

  Clarke, McMillan, Somenzi, Biere, Cimatti, ...

Beyond the finite-state case

- Unbounded (parametric/dynamic) number of threads
  - Undecidable in general if threads Ids are allowed
  - $\Rightarrow$ Anonymous threads
- Unbounded channels
  - Undecidable in general in case of FIFO queues
  - $\Rightarrow$ Unordered queues (multisets), lossy queues

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- Safety is reducible to state reachability in VASS / Coverability in PN

Vector Addtion Systems with States

- Finite state machine + finite number of counter $C = \{c_1, \ldots, c_n\}$.
- Operations: (No test to zero)
  - $c_i := c_i + 1$
  - $c_i > 0 \;/\; c_i := c_i - 1$
- Configuration: (q, V) where q is a control state and $V \in \mathbb{N}^n$
- Initial configuration: $(q_0, \mathbf{0})$ where $\mathbf{0} = 0^n$.
- Transition relation:

$$(q_1, V_1) \xrightarrow{op} (q_2, V_2) \textit{ iff}$$

  - $op =$ "$c_i := c_i + 1$", *and* $V_2 = V_1[c_i \leftarrow (V_1(c_i) + 1)]$
  - $op =$ "$c_i > 0 \;/\; c_i := c_i - 1$, *and* $(V_1(c_i) > 0$ *and* $V_2 = V_1[c_i \leftarrow (V_1(c_i) - 1)])$

From Multithreaded Programs to VASS

- Associate a control state with each valuation of the globals
- Associate a counter with each valuation of thread locals
- A statement moving globals from g to g' and locals from ℓ to ℓ':

$$g \xrightarrow{c_\ell > 0 / c_\ell := c_\ell - 1 \,;\, c_{\ell'} := c_{\ell'} + 1} g'$$

- Creation of a new thread at initial state ℓ:

$$g \xrightarrow{c_\ell := c_\ell + 1} g$$

VASS: State Reachability

- State reachability problem:

  Given a state q, determine if a configuration (q, V) is reachable, for some $V \in \mathbb{N}^n$ (any one).

- Coverability problem:

  Given a configuration (q, V), determine if a configuration (q, V') is reachable, for some $V' \geq V$. (We say that (q, V) is coverable*.)*

EXSPACE-complete [Rackoff 78]

NB: *Coverability can be reduced to State Reachability and vice-versa.*

Well Structured Systems

[Abdulla et al. 96], [Finkel, Schnoebelen, 00]

- Let U be a universe.
- Well-quasi ordering $\preceq$ over U: $\forall c_0, c_1, c_2, \ldots, \ \exists i < j, \ c_i \preceq c_j$
- $\Rightarrow$ Each (infinite) set has a finite minor set.

- Let $S \subseteq U$. Upward-closure $\overline{S}$ = minimal subset of U s.t.
  - $S \subseteq \overline{S}$,
  - $\forall x, y.\ (x \in S \text{ and } x \preceq y) \Rightarrow y \in \overline{S}$.
- A set is upward closed if $\overline{S} = S$
- Upward closed sets are definable by their minor sets
  - Assume there is a function *Min* which associates a minor to each set.
  - Assume $pre(Min(S))$ is computable for each set S.

- Monotonicity: $\preceq$ is a simulation relation

$$\forall c_1, c_1', c_2.\ \big((c_1 \longrightarrow c_1' \text{ and } c_1 \preceq c_2) \Rightarrow \exists c_2'.\ c_2 \longrightarrow c_2' \text{ and } c_1' \preceq c_2'\big)$$

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2. Assume $pre(S)$ is not upward closed.
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6. S is upward closed $\Rightarrow$ $c_2' \in S$

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7. $\Rightarrow c_2 \in pre(S)$, contradiction.

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7. $\Rightarrow c_2 \in pre(S)$, contradiction.
8. For pre^*: the union of upward closed sets is upward closed.

Backward Reachability Analysis

Consider the increasing sequence $X_0 \subseteq X_1 \subseteq X_2 \ldots$ defined by:

- $X_0 = Min(S)$
- $X_{i+1} = X_i \cup Min(pre(\overline{X_i}))$

Termination:

There is a index $i \geq 0$ *such that* $X_{i+1} = X_i$

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- How long shall we wait?
- Possibly very very long: Non primitive recursive in general

The case of VASS

- Usual $\leq$ order over $\mathbb{N}$ is a WQO (Dickson lemma)
- Product of WQO's is a WQO.
- $\Rightarrow \leq$ generalized to $\mathbb{N}^n$ is a WQO.

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- Upward-closed sets = finite disjunctions of $\bigwedge_{i=1}^{n} l_i \leq c_i$, where $l_i \in \mathbb{N}$
- Computation of the Pre:
  - $op =$ "$c_j := c_j + 1$" : $(\bigwedge_{i \neq j} l_i \leq c_i) \wedge (max(l_j - 1, 0) \leq c_j)$
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- No test to zero, only guards of the form $c > 0 \Rightarrow$ Monotonicity
- $\Rightarrow$ Coverability is decidable.

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- $\Rightarrow$ Restrictions
  - Classes of programs with particular features
  - Particular kind of behaviors
    (under-approximate analysis for bug detection)

Asynchronous Programs

- Synchronous calls

  Usual procedure calls

- Asynchronous calls
  - *Calls are stored and dispatched later by the scheduler*
  - *They can be executed in any order*

- Event-driven programming (requests, responses)
- Useful model: distributed systems, web servers, embedded systems

Formal Models: Multiset Pushdown Systems

- A task is a sequential (pushdown) process with dynamic task creation
- Created tasks are stored in an unordered buffer (multiset)
- Tasks run until completion
- If the stack is empty, a task in moved from the multiset to the stack

Difficulties

- Unbounded buffer of tasks
- The buffer is a multiset $\Rightarrow$ can be encoded as counters
- Need to combine somehow PDS with VASS
- Stack $\Rightarrow$ not Well Structured
- How to get rid of the stack ?

State Reachability of Multiset PDS

Theorem

The control state reachability problem for MPDS is EXPSPACE-complete.

Reduction to/from the coverability problem for Petri.

First decidability proof by K. Sen and M. Viswanathan, 2006

Semi-linear Sets

- Linear set over $\mathbb{N}^n$ is a set of the form

$$\{\vec{u} + k_1\vec{v_1} + \cdots + k_m\vec{v_m} \ : \ k_1, \ldots, k_m \in \mathbb{N}\}$$

  where $\vec{u}, \vec{v_1}, \ldots, \vec{v_m} \in \mathbb{N}^n$

- Semi-linear set = finite union of linear sets.

- Examples:
  - $\{(0,0) + k(1,1) \ : \ k \geq 0\} \ \equiv \ x_1 = x_2$
  - $\{(0,0) + k(1,2) \ : \ k \geq 0\} \ \equiv \ 2x_1 = x_2$
  - $\{(0,3) + k(1,1) \ : \ k \geq 0\} \ \equiv \ x_1 + 3 = x_2$
  - $\{(0,3) + k_1(0,1) + k_2(1,1) \ : \ k \geq 0\} \ \equiv \ x_1 + 3 \leq x_2$
  - $\{(0,0,0) + k_1(1,0,1) + k_2(0,1,1) \ : \ k_1, k_2 \geq 0\} \ \equiv \ x_1 + x_2 = x_3$
  - $\{(0,0,3) + k_1(1,0,2) + k_2(0,1,1) \ : \ k_1, k_2 \geq 0\} \ \equiv \ 2x_1 + x_2 + 3 = x_3$

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- Theorem [Ginsburg, Spanier, 1966]

  A set is semi-linear iff it is definable in Presburger arithmetics.

Parikh's image

- Let $\Sigma = \{a_1, \ldots, a_n\}$.
- Given a word $w \in \Sigma^*$, the *Parikh image* of w is:

$$\phi(w) = (\#_{a_1}(w), \ldots, \#_{a_n}(w)) \in \mathbb{N}^n$$

- Given a language $L \subseteq \Sigma^*$, $\phi(L) = \{\phi(w) \ : \ w \in L\}$
- Examples:
  - $L_1 = \{a^n b^n \ : \ n \geq 0\}$, $\phi(L_1) = \{(x_1, x_2) \ : \ x_1 = x_2\}$
  - $L_2 = \{a^n b^n c^n \ : \ n \geq 0\}$, $\phi(L_2) = \{(x_1, x_2, x_3) \ : \ x_1 = x_2 \land x_2 = x_3\}$
  - $L_3 = (ab)^* = \{(ab)^n \ : \ n \geq 0\}$, $\phi(L_3) = \{(x_1, x_2) \ : \ x_1 = x_2\}$

Semi-linear sets, CFL's, and RL's

- Parikh's Theorem (1966)

  For every Context-Free Language L, $\phi(L)$ *is a semi-linear set.*

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  For every Context-Free Language L, $\phi(L)$ is a semi-linear set.

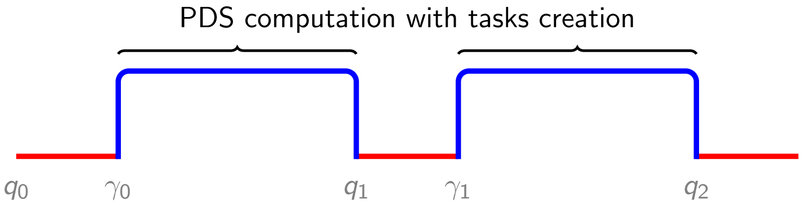
- Proposition

  For every semi-linear set S, there exists a Regular Language L such that $\phi(L) = S$.

- Corollary

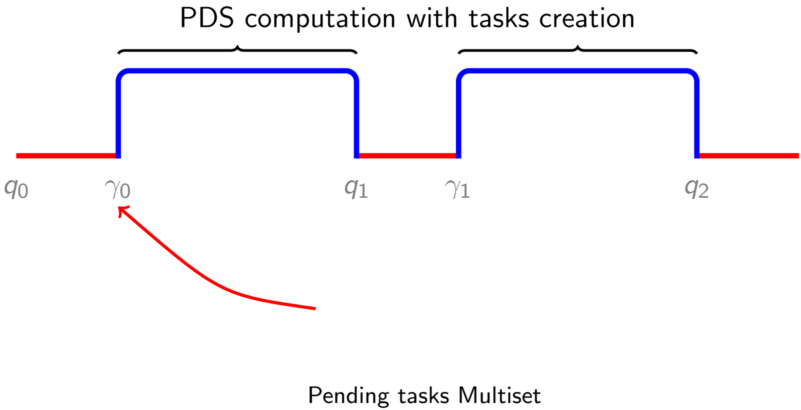
  For every Context-Free Language L, there exists a Regular language L' such that $\phi(L) = \phi(L')$.

From Multiset PDS to VASS



Pending tasks Multiset

From Multiset PDS to VASS



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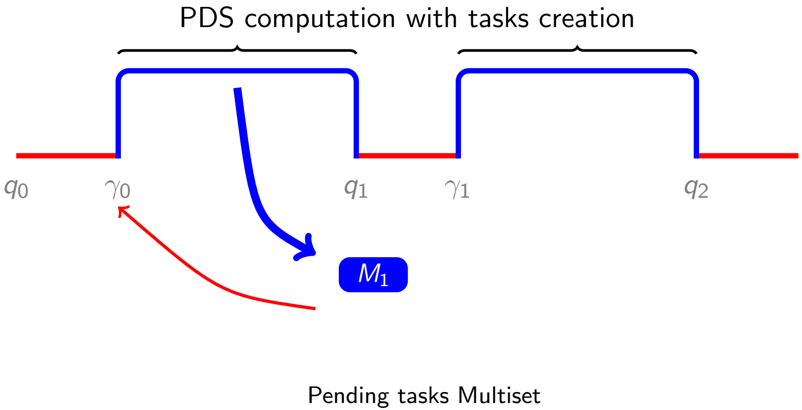

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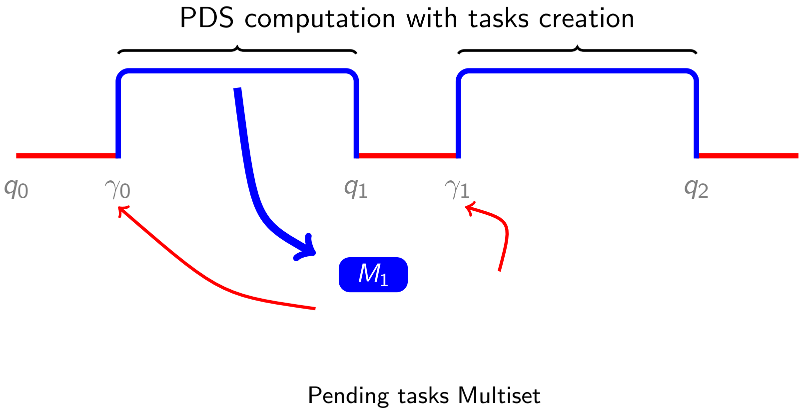

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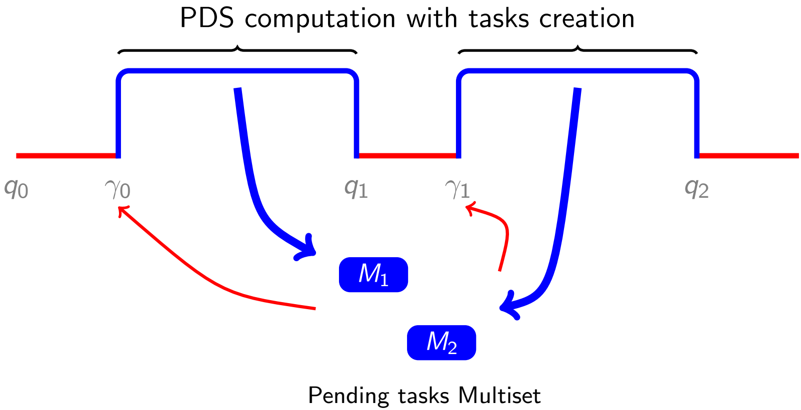

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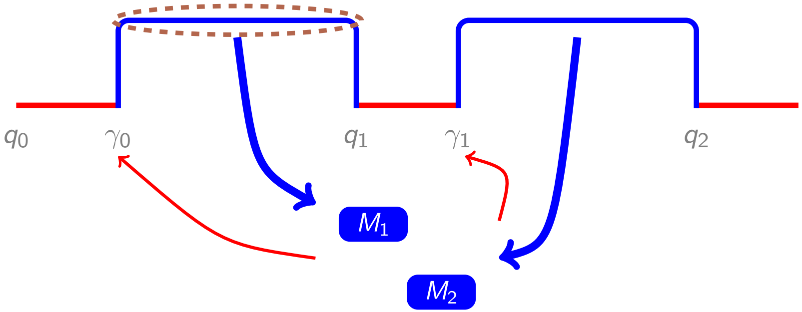

$$q_0, \gamma_0 \overset{L_1}{\Longrightarrow}^* q_1, \epsilon$$

L_1 = Set of sequences of created tasks

L_1 is a Context-Free Language

M_1 is the Parikh image of L_1

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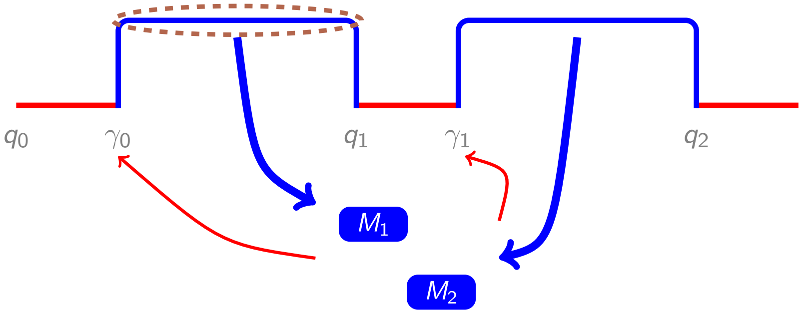

Parikh's Theorem: M_i is definable by a finite state automaton S_i

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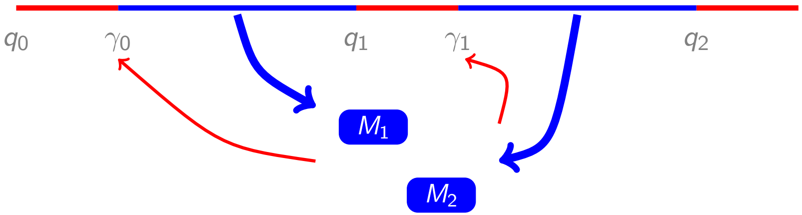

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Construction of a VASS: Simulation of S_i + task consumption rules

Message-Passing Programs with Procedures

- Undecidable even for unbounded FIFO channels
- Restrictions on
  - Interaction between recursion and communication (e.g., communication with empty stack)
  - Kind of channels (e.g., lossy, unordered)
  - Topology of the network
- Decidable classes

  [La Torre et al. TACAS'08], [Atig et al., CONCUR'08], ...

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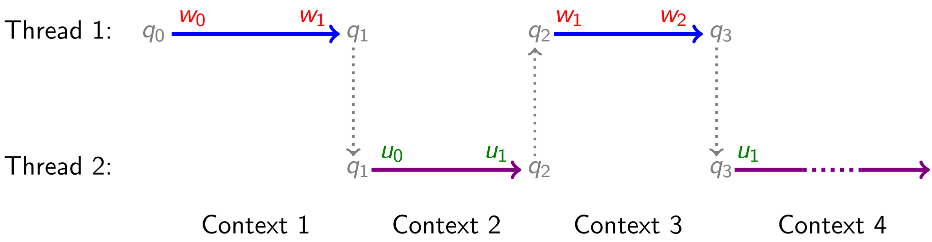
- What is a good concept for restricting the set of behaviors ?

Context-Bounded Analysis

[Qadeer, Rehof, 2005]

The number of context switches in a computation is bounded



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- Concurrency bugs show up after a small number of context switches.

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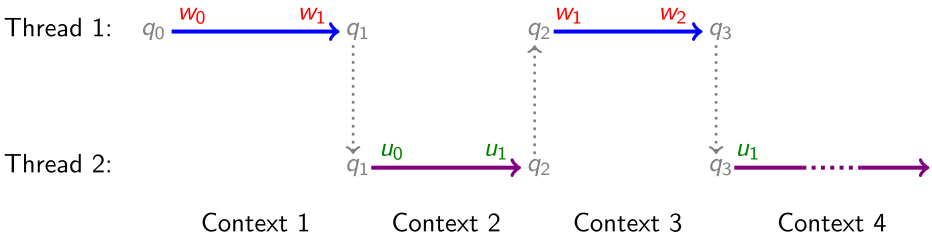

- Suitable for finding bugs in concurrent programs.
- Concurrency bugs show up after a small number of context switches.
- Infinite-state space: Unbounded sequential computations
- Decidability ?

Basic case: Pushdown system

- Pushdown system $= (Q, \Gamma, \Delta)$
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- Computation of the predecessors/successors:

  For every regular set of configurations C, the $pre^*(C)$ and $post^*(C)$ are regular and effectively constructible.
  [Büchi 62], ..., [B., Esparza, Maler, 97], ...

- Reachability: Polynomial algorithms.
- Can be generalized to model checking.

Context-Bounded Analysis: Decidability

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- Enumerate sequences of the form $q_0 i_0 q_1 i_1 q_2 i_2 \ldots i_K q_K i_{K+1}$, where q_j's are states, and $i_j \in \{1, \ldots, n\}$ are threads identities.
- Let $X_{K+1} = C$. Compute: for $j = K$ back to 0
  - $A'_{j+1} = pre^*_{i_{j+1}}(X_{j+1}[i_{j+1}]) \cap q_j \Gamma_i^*$
  - $X_j = (q_j, A_1^{j+1}, \ldots, A'_{j+1}, \ldots, A_n^{j+1})$

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Is it possible to reduce CBA of a Concurrent Program to the Reachability Analysis of a Sequential Program ?

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Yes: Use compositional reasoning !

[Lal, Reps, 2008]

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- Check that the interfaces are composable
  - $O^1_j = I^2_j$ for every $j \in \{1, \ldots, K\}$
  - $O^2_j = I^1_{j+1}$ for every $j \in \{1, \ldots, K-1\}$

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- Create $2K$ copies of the global variables X_j and X'_j, for $j \in \{1, \ldots, K\}$
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  1. Assign $*$ to all variables of X_j (guesses the input I^1_j)
  2. Copies X_j in X'_j, and runs by using X'_j as global variables
  3. Choses nondeterministically the next context-switch point
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  4. Checks that $X'_j = X_{j+1}$ (composability check), and move to round $j+1$
  5. If q_2 is reachable at round K, then go to state q_{win}

Dynamic Creation of Threads ?

[Atig, B., Qadeer, 09]

Problem

- Bounding the number of context switches $\Rightarrow$ bounding the number of threads.
- $\Rightarrow$ Inadequate bounding concept for the dynamic case.

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New definition

- Give to each thread a *context switch budget*
- $\Rightarrow$ The number of context switches is bounded for each thread
- $\Rightarrow$ The global number of context switches in a run is unbounded
- NB: Generalization of Asynchronous Programs

Case 1: Dynamic Networks of Finite-State Processes

Decidable ?

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Theorem

The K-bounded state reachability problem is EXPSPACE-complete.

Reduction to/from the coverability problem for Petri.

Reduction to coverability in PN

- For every global store $q \in Q$, associate a place q.
- For every stack configuration $\gamma \in \Gamma \cup \{\epsilon\}$ and budget $b \in \{1, \ldots, K\}$ of the active thread, associate a place $(\gamma,\mathrm{b},\mathrm{Act})$.
- For every stack configuration $\gamma \in \Gamma \cup \{\epsilon\}$ and budget $b \in \{0, \ldots, K\}$ of a pending thread, associate a place $(\gamma,\mathrm{b},\mathrm{Pen})$.

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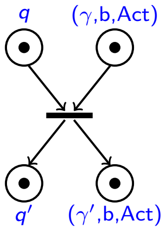
Rule of the form: $q\gamma \longrightarrow q'\gamma'$ $\Longrightarrow$



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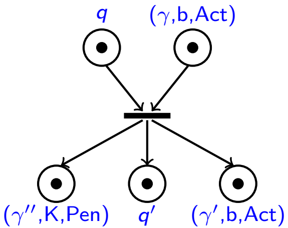

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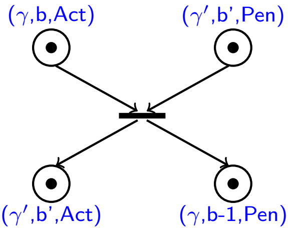
Context switch (with b'$>$ 0) $\Longrightarrow$



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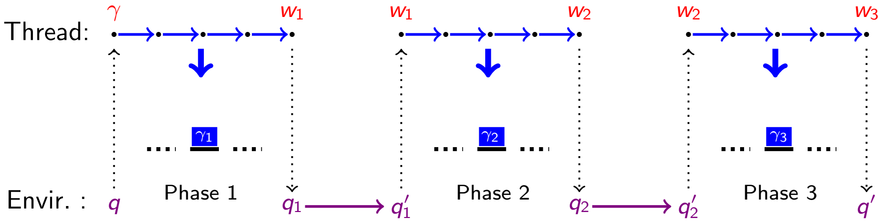
Theorem

The K-bounded state reachability problem is in 2EXPSPACE.

Exponential reduction to the coverability problem in PN

Making visible the interactions



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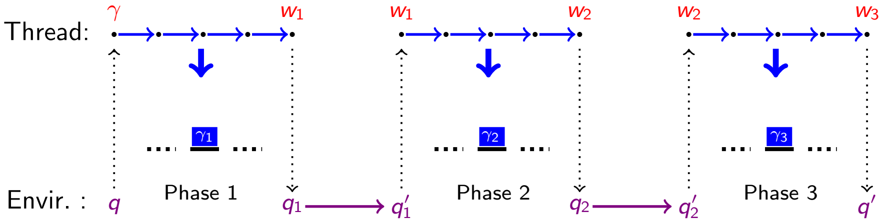

- Construct a labeled pushdown automaton which:
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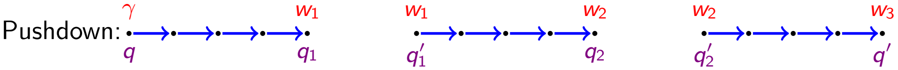

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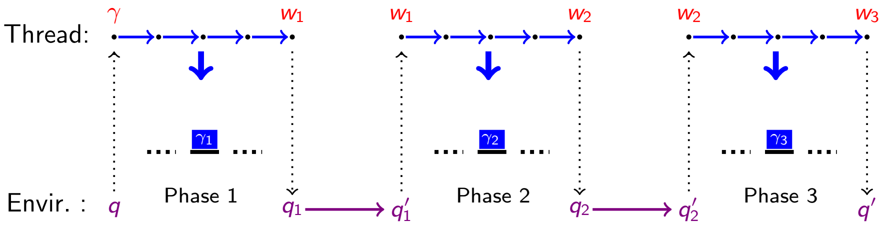

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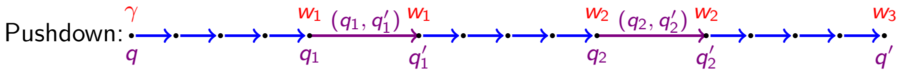

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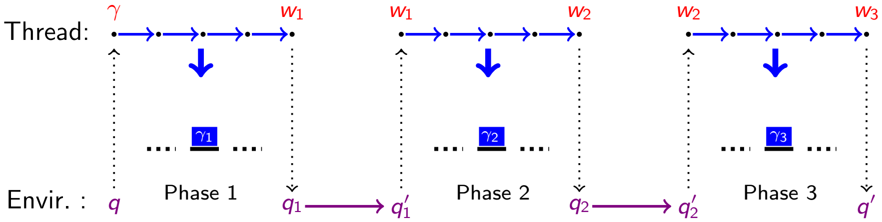

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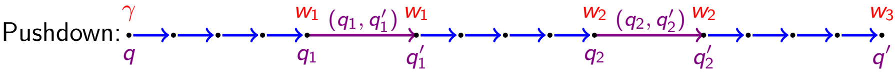

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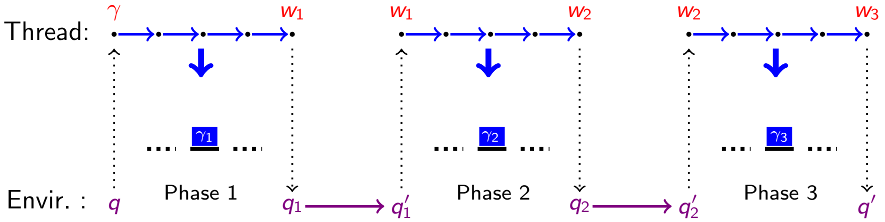

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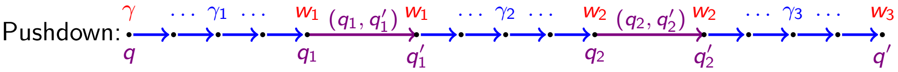
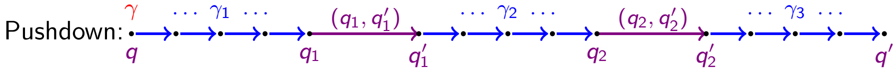


Constructing a regular interface



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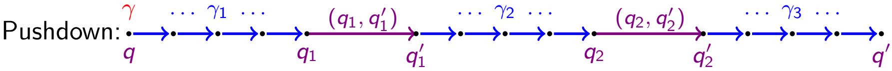

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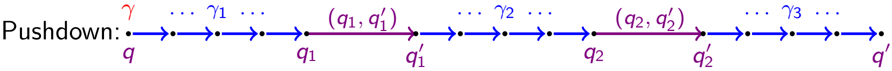

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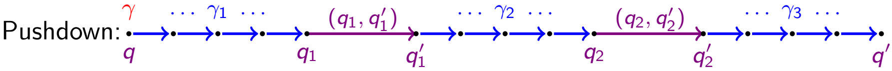

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$\Rightarrow$ Replace L by its downward closure w.r.t. the sub-word relation $L\downarrow$

Constructing a regular interface (cont.)

- The interactions of a thread with its environment can be characterized by the downward closure $L\downarrow$ of the context-free language L
- $L\downarrow$ is regular and effectively constructible ([Courcelle, 1991])
- The size of an automaton for $L\downarrow$ can be exponential in the PDA defining L

Constructing the Petri Net

- Use places for representing the control, one per state
- Count pending tasks having some context switch budget (from 0 to K), and waiting to start at some state
- For each created task, guess a sequence of K states (for context switches)
- At context switches, control is given to a pending task waiting for the current state
- Simulate a full sequential computation (following the FSA automaton of the interface) until next transition (g, g')
- During the simulation, each transition labelled γ corresponds to a task creation
- At a transition (g, g'), leave the control at g (to some other thread) and wait for g' (with a lower switch budget)

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- Under-approximate sequentialization [B., Emmi, Parlato, 2011]
- Idea:
  - Transform thread creation into procedure calls
  - Allow some reordering using the idea of bounded interfaces

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- Infinite behaviors (liveness bugs):
  - K-context-bounded ultimately periodic behaviors

    [Atig, B., Emmi, Lal, CAV'12]
  - Scope-bounded analysis

    [LaTorre, Napoli, CONCUR'11], [Atig, B., N. Kumar, Saivasan, ATVA'12]