Visibly Pushdown Automata with Multiplicities: Finiteness and K-Boundedness

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Outline

Visibly Pushdown Automata (VPA)

- Subclass of Pushdown automata,
- Partitioned input alphabet: $\Sigma = \Sigma_c \uplus \Sigma_r \uplus \Sigma_\iota$,
- On Σ_c (call): push, on Σ_r (return): pop and on Σ_ι (internal): nothing.

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Definition (R. Alur and P. Madhusudan 2004)

A visibly pushdown automaton on Σ is a tuple $(\Gamma, Q, Q_{in}, Q_f, \delta)$ with:

- Γ is the set of stack symbols,
- Q is the set of states,
- $Q_{in} \subseteq Q$ (resp. $Q_f \subseteq Q$) is the set of initial (resp. final) states,
- $\delta \subseteq (Q \times \Sigma_\iota \times Q) \cup (Q \times \Sigma_c \times \Gamma \times Q) \cup (Q \times \Sigma_r \times \Gamma \times Q)$ is the set of transitions.

Well-nested words

Σ^*_{wn} is a subset of Σ^* such that:

- $\Sigma^*_\iota \subseteq \Sigma^*_{wn}$
- $c \in \Sigma_c$, $r \in \Sigma_r$ and $u \in \Sigma^*_{wn}$ then $cur \in \Sigma^*_{wn}$.
- $u_1, u_2 \in \Sigma^*_{wn}$, then $u_1 u_2 \in \Sigma^*_{wn}$.

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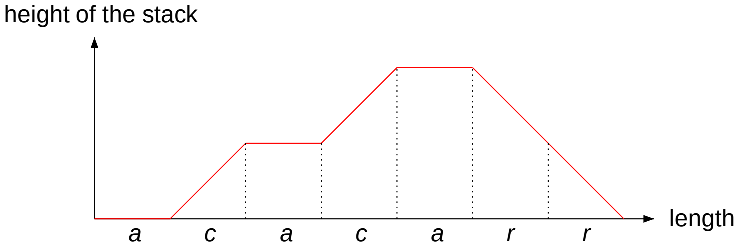
- $u_1, u_2 \in \Sigma^*_{\text{wn}}$, then $u_1u_2 \in \Sigma^*_{\text{wn}}$.

For well-nested word $w = acacarr$, with $a \in \Sigma_\iota, c \in \Sigma_c, r \in \Sigma_r$:



Runs and accepting runs

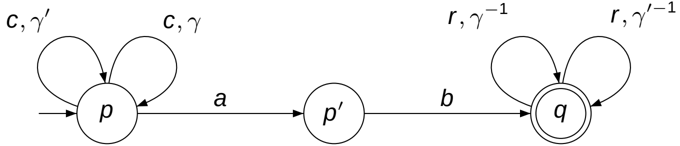
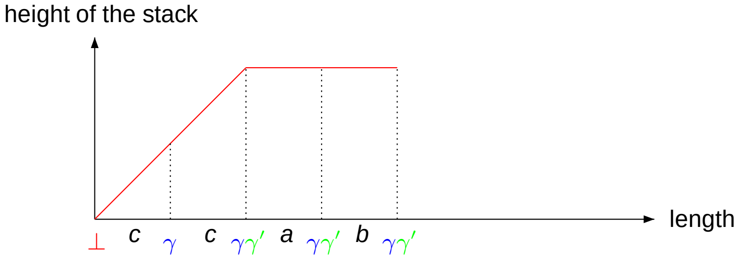


Figure: $\mathcal{L}(A) = \{c^n a b r^n \mid n \in \mathbb{N}\}$, $a, b \in \Sigma_\iota, c \in \Sigma_c, r \in \Sigma_r$

Run: $(p, \bot) \xrightarrow{c,\gamma} (p, \gamma) \xrightarrow{c,\gamma'} (p, \gamma\gamma') \xrightarrow{a} (p', \gamma\gamma') \xrightarrow{b} (q, \gamma\gamma')$ (on word $w = ccab$)



Runs and accepting runs

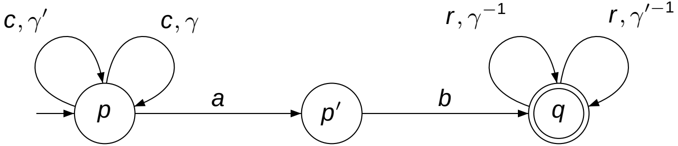
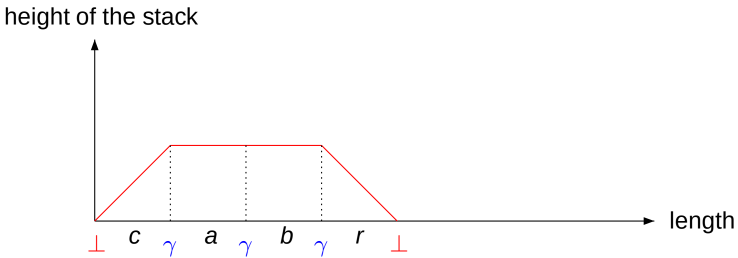


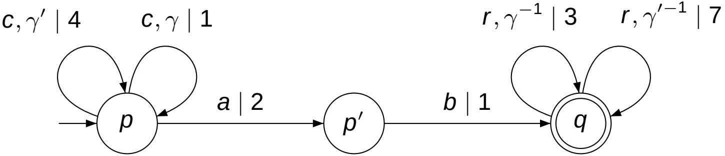
Figure: $\mathcal{L}(A) = \{c^n abr^n \mid n \in \mathbb{N}\}$, $a, b \in \Sigma_\iota, c \in \Sigma_c, r \in \Sigma_r$

Accepting run: $(p, \bot) \xrightarrow{c,\gamma} (p, \gamma) \xrightarrow{a} (p', \gamma) \xrightarrow{b} (q, \gamma) \xrightarrow{r,\gamma^{-1}} (q, \bot)$ (on word $w = cabr$)



$\mathbb{N}$-VPA

Visibly pushdown automata with cost function $\lambda : \delta \rightarrow \mathbb{N}_{>0}$,



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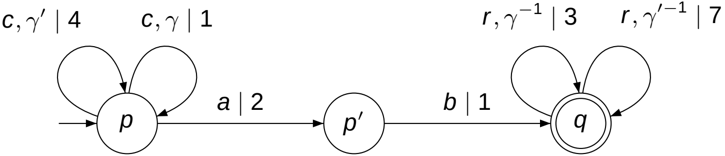

Figure: $\mathcal{L}(A) = \{c^n a r^n \mid n \in \mathbb{N}\}$, $a, b \in \Sigma_\iota, c \in \Sigma_c, r \in \Sigma_r$

- Multiplicity of a run ρ over transitions $\{t_i\}_{1 \le i \le k}$: $\langle \rho \rangle = \prod_{i=1}^{k} \lambda(t_i)$,
  $\rho : (p, \perp) \xrightarrow{c,\gamma'} (p, \gamma') \xrightarrow{a} (p', \gamma') \xrightarrow{b} (q, \gamma') \xrightarrow{r,\gamma'^{-1}} (q, \perp)$, $\langle \rho \rangle = 56$
- Multiplicity of a word u: $\langle u \rangle = \sum_{\rho \in acceptingRuns(u)} \langle \rho \rangle$,
  $u = cabr$, $\langle u \rangle = 62$
- Multiplicity of a $\mathbb{N}$-VPA A: $\langle A \rangle = sup_{u \in \mathcal{L}(A)} \langle u \rangle$.

Outline

Pattern **(S1)**

We give a characterization on $\mathbb{N}$-VPA ensuring their infiniteness by means of patterns (dumbbell):

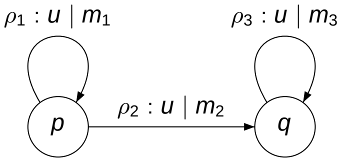

Figure: **(S1)** Well-nested case: $u \in \Sigma^*_{\text{wn}}$. Either **a)** $m_1 > 1$ or **b)** $\rho_1 \neq \rho_2$.

Lemma

Let A be an $\mathbb{N}$-VPA, then if A complies with **(S1)**, $\langle A \rangle = +\infty$.

If A complies with **(S1.a)**, then $\langle A \rangle = +\infty$

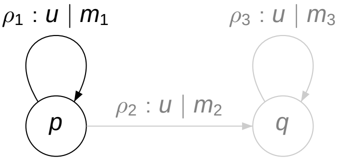

Figure: $u \in \Sigma_{\text{wn}}^*, m_1 > 1$

$\langle u^i \rangle = \langle \rho_1 \rangle^i = m_1^i$.

$\exists w w' \in \Sigma_{\text{wn}}^*$ such that

$$(q_{in}, \perp) \xrightarrow{w} (p, \sigma) \xrightarrow{u^i} (p, \sigma) \xrightarrow{w'} (q_f, \perp) \text{ is an accepting run of } A.$$

Then $\langle w u^i w' \rangle \geq m_1^i \underset{i \to +\infty}{\longrightarrow} +\infty$.

If A complies with **(S1.b)**, then $\langle A \rangle = +\infty$

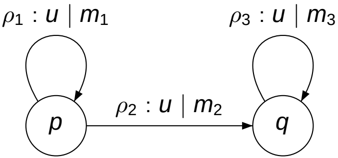

Figure: $u \in \Sigma_{\text{wn}}^*, \rho_1 \neq \rho_2$

$\langle u^3 \rangle = \langle \rho_1\rho_2\rho_3 \rangle + \langle \rho_1\rho_1\rho_2 \rangle + \langle \rho_2\rho_3\rho_3 \rangle = m_1m_2m_3 + m_1m_1m_2 + m_2m_3m_3$.
Then $\langle u^i \rangle = \sum_{j=0}^{i-1} \langle \rho_1^j \rho_2 \rho_3^{i-j-1} \rangle$.

$\exists ww' \in \Sigma_{\text{wn}}^*$ such that

$(q_{in}, \perp) \xrightarrow{w} (p, \sigma) \xrightarrow{u^j} (p, \sigma) \xrightarrow{u} (q, \sigma) \xrightarrow{u^{i-j-1}} (q, \sigma) \xrightarrow{w'} (q_f, \perp)$ is an accepting run of A.

Then $\langle wu^iw' \rangle \geq i \underset{i \to +\infty}{\longrightarrow} +\infty$.

Pattern **(S2)**

We give a characterization on $\mathbb{N}$-VPA ensuring their infiniteness by means of patterns (dumbbell):

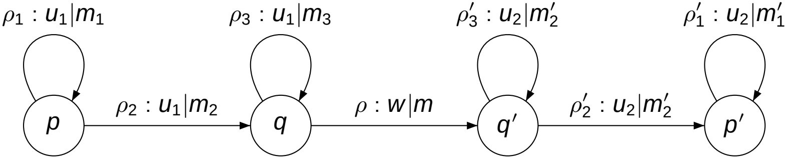

Figure: **(S2)** Matched loops case: $w \in \Sigma^*_{\mathrm{wn}}$, $u_1u_2 \in \Sigma^*_{\mathrm{wn}}$, and $u_1 \notin \Sigma^*_{\mathrm{wn}}$. Either **a)** ($m_1 > 1 \vee m'_1 > 1$), or **b)** ($\rho_1 \neq \rho_2 \vee \rho'_1 \neq \rho'_2$).

Lemma

Let A be an $\mathbb{N}$-VPA. If A complies with **(S2)**, $\langle A \rangle = +\infty$.

Results

The presence of **(S1)** or **(S2)** is a sufficient condition to the infinite multiplicity of a $\mathbb{N}$-VPA...

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Theorem

Let A be an $\mathbb{N}$-VPA. $\langle A \rangle = +\infty$ if and only if A complies with one of the criteria **(S1)** and **(S2)**.

Theorem

Let A be a $\mathbb{N}$-VPA. It is decidable in polynomial time whether $\langle A \rangle = +\infty$.

Outline

Results

Theorem

Given an $\mathbb{N}$-VPA A and $k \in \mathbb{N}_{>0}$, the problem of determining whether $\langle A \rangle < k$ is EXPTIME-complete.

Theorem

For a fixed $k \in \mathbb{N}_{>0}$, given an $\mathbb{N}$-VPA A the problem of determining whether $\langle A \rangle < k$ is in PTIME.

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Conclusion

- Complete characterization of infinite multiplicity of $\mathbb{N}$-VPA.
- Deciding the k-boundedness of $\mathbb{N}$-VPA.

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- Deciding the k-boundedness of $\mathbb{N}$-VPA.

- Valuedness for visibly pushdown transducers.