-MOVEP 2012-

Implementation of Timed Systems: Theory and Practice

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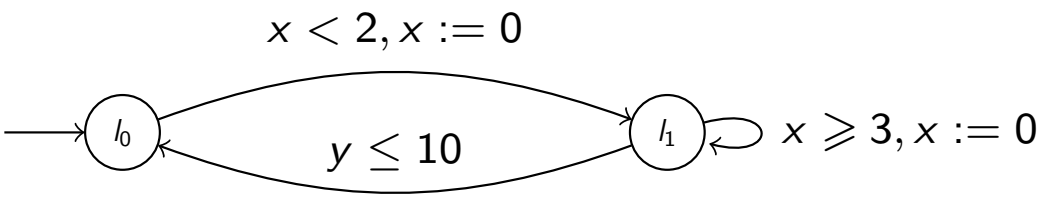
December 4, 2012

Outline

Parametric Approach in Automata Theory

Timed Automaton (TA)

- Finite automaton extended with a finite set of clocks



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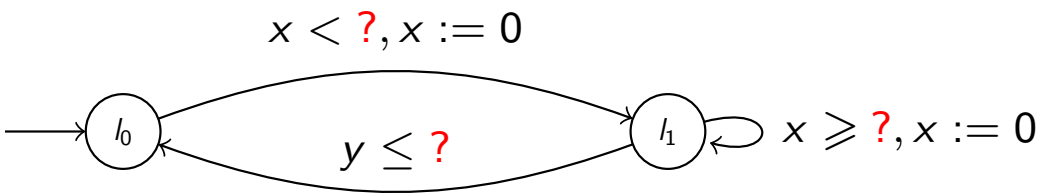

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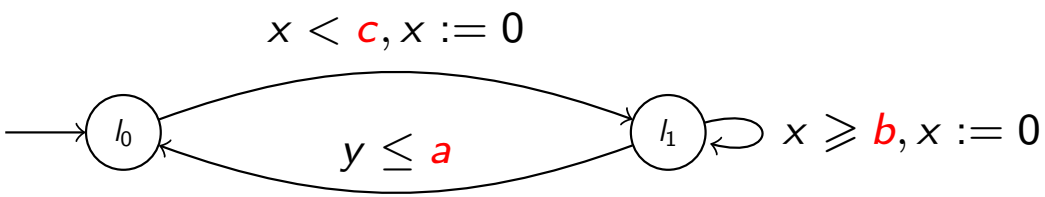

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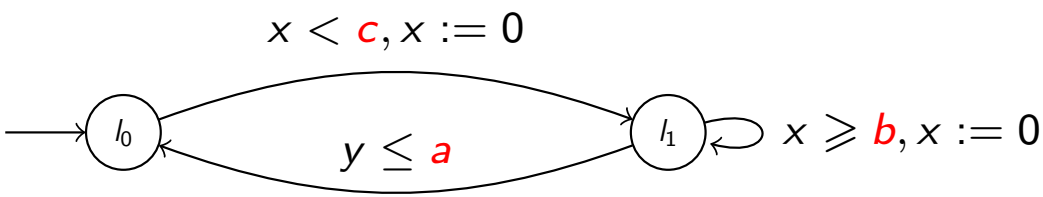

Parametric Timed Automaton (PTA)

- Timed automata with parameters as bounds on clocks

Challenges of the Parametric Approach

Parametric verification

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Synthesis problem

Can we compute all of these valuations?

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Undecidability Results

Theorem — Alur & Dill '93

Reachability emptiness problem is undecidable for PTA.

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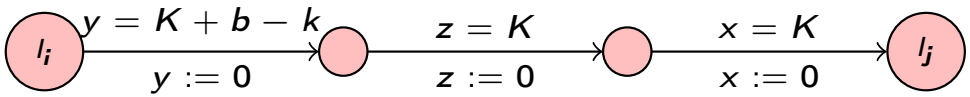
- New encoding of the 2-counter machine with parameters bounded by K and k:



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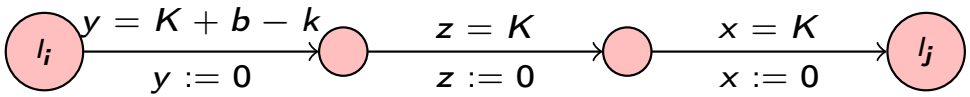

Theorem

Reachability emptiness problem with bounded parameters is undecidable for PTA.

Undecidability Results *cont.*

Subclass of PTA: L/U automaton [Hune & al. '02]:

- Two sets of parameters: lower bounds on clocks ($x \geq a$) upper bounds on clocks ($x \leq b$)
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- transform a PTA into a L/U automaton
- we obtain: $v_p \Leftrightarrow v_{L/U} \cap \{p_i^l = p_i^u\} \stackrel{?}{=} \{\emptyset\} \implies$ undecidable

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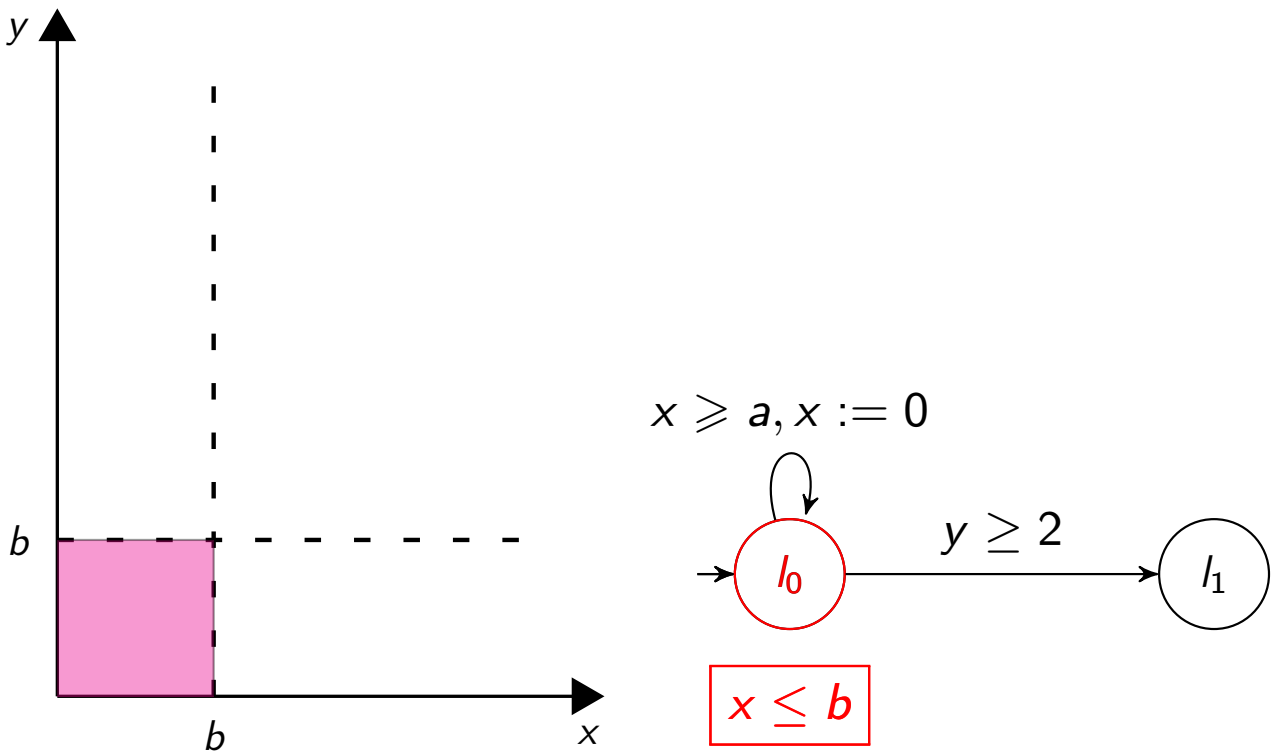
Theorem

The solution to the reachability emptiness problem for L/U automata cannot be represented using any formalism for which the emptiness of the intersection with equality constraints is decidable.

Motivating example

Forward state-space exploration of PTA

- Condition for termination: $Z_{n+1} \subseteq Z_n$



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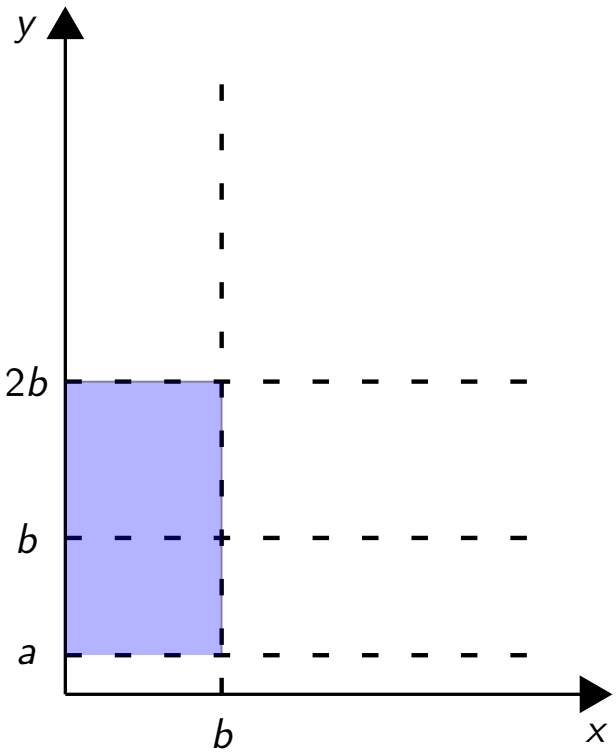

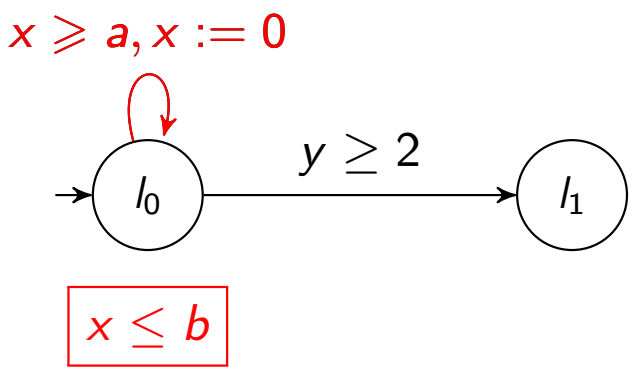

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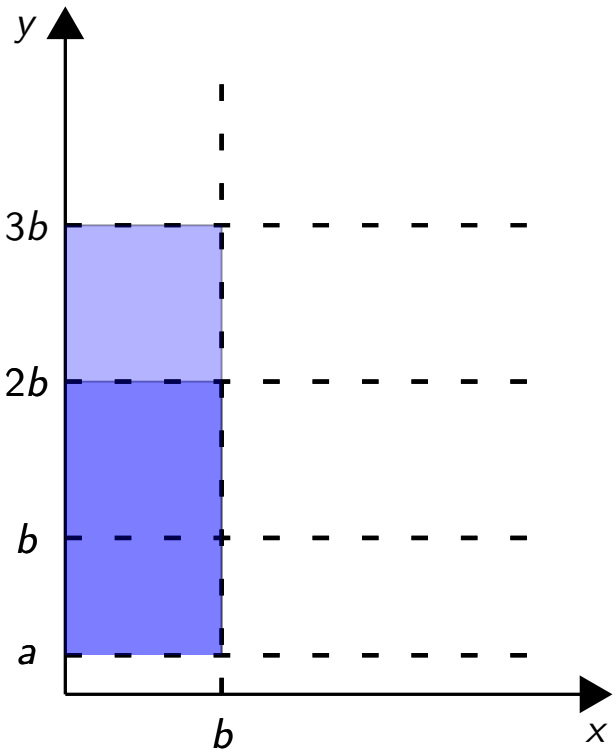

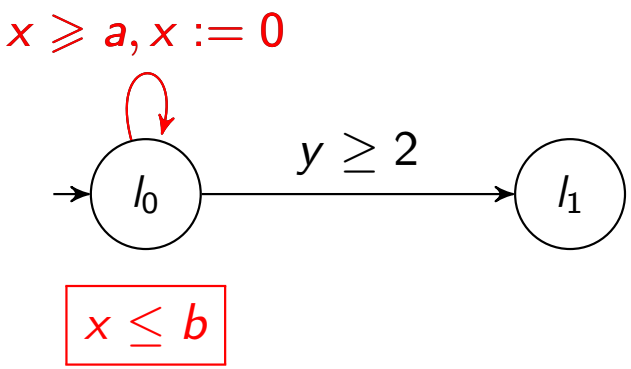

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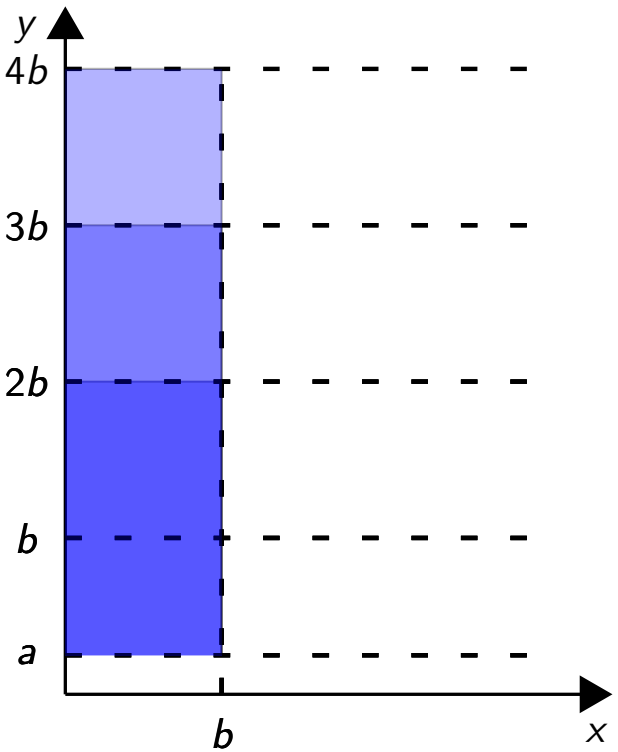

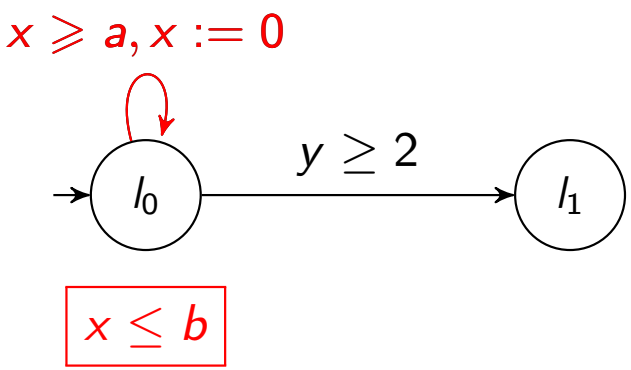

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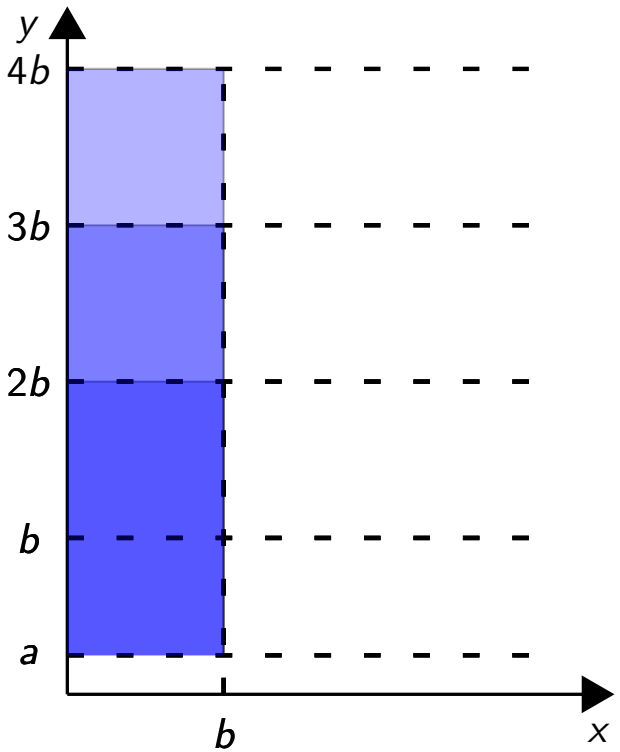

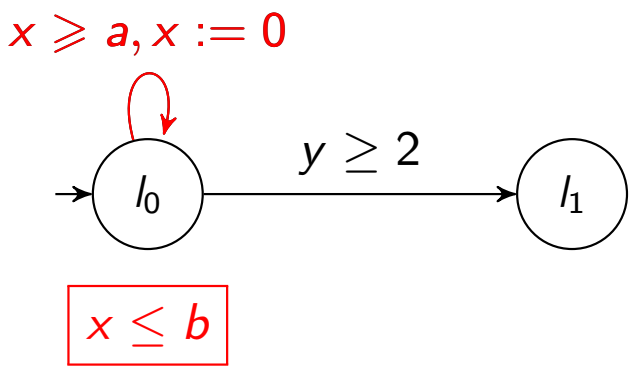

- $Z_n = \{0 \leq x \leq b, 0 \leq na \leq y \leq (n+1)b\}$

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Integer Parameter Synthesis for PTA

Current problem:

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Proposed solution:

- computation of integer parameter valuations
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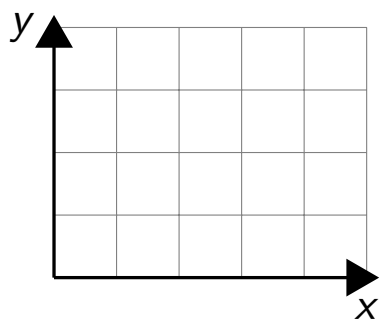

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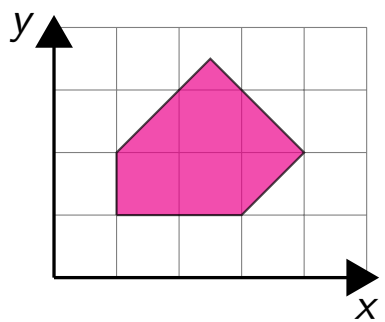

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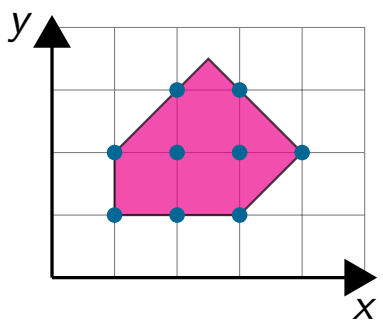

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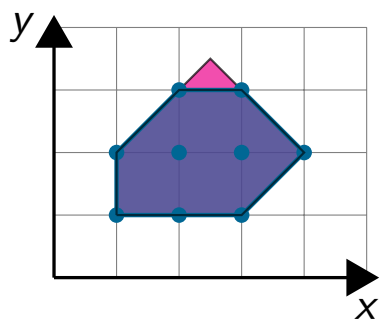

IntHull(Z): smallest subset of elements of Z with *integer coordinates*

Termination

Problem: termination of the algorithm

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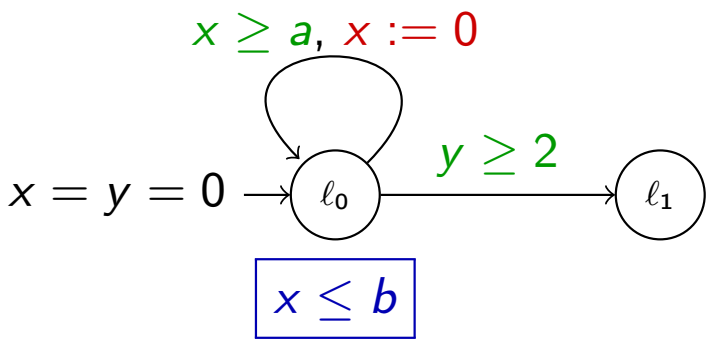
- Clocks are *upper bounded* by N



- Suppose $N = 3$ and (ℓ_0, Z_0) with $Z_0 = \{x = y, x \leq b \leq 3\}$;
- After one loop: $Z_1' = Z_1 \cap \{y \leq b + 1\}$;
- After two loops: $Z_2' = Z_2 \cap \{a \leq 1 \text{ and } a \leq b\}$
- After $n > 3$ loops:

$$Z_n' = Z_{n+1}' = \{a = 0, 0 \leq x \leq b, x \leq 3, 0 \leq y \leq 3b, y \leq 3\}$$

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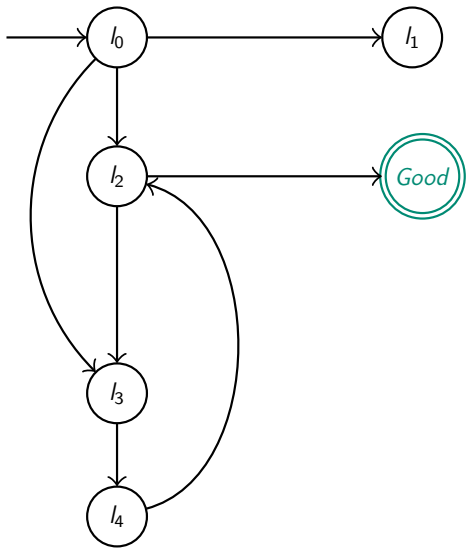
Parametric Timed Game Automata

Control problems on TA:



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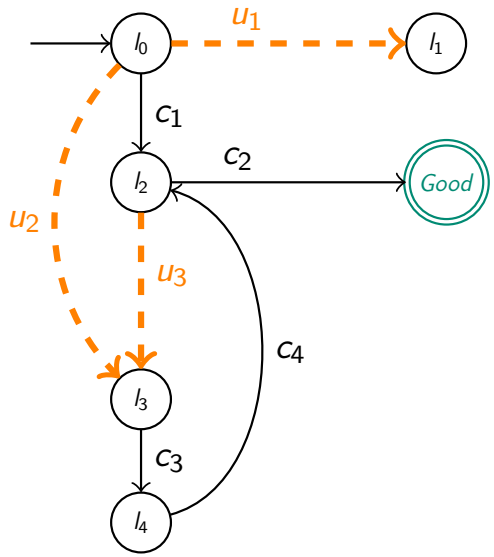

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Control problems on TA:

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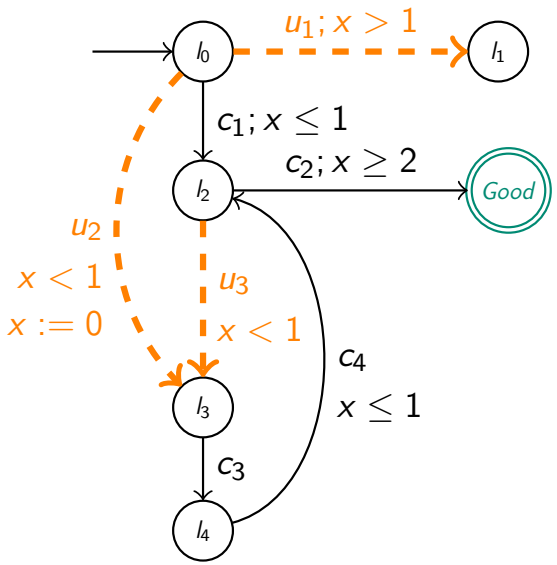

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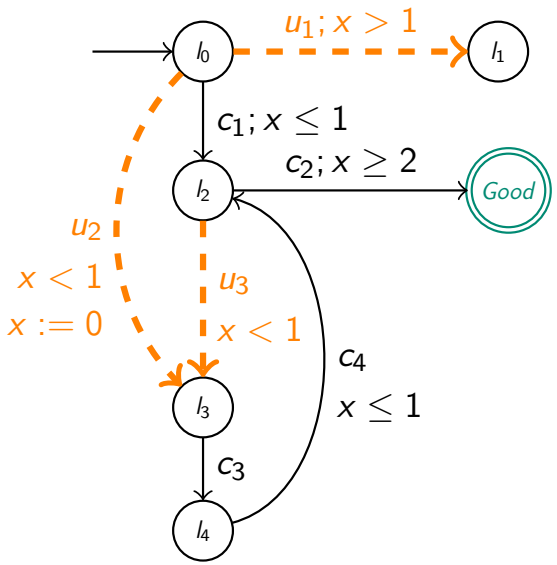

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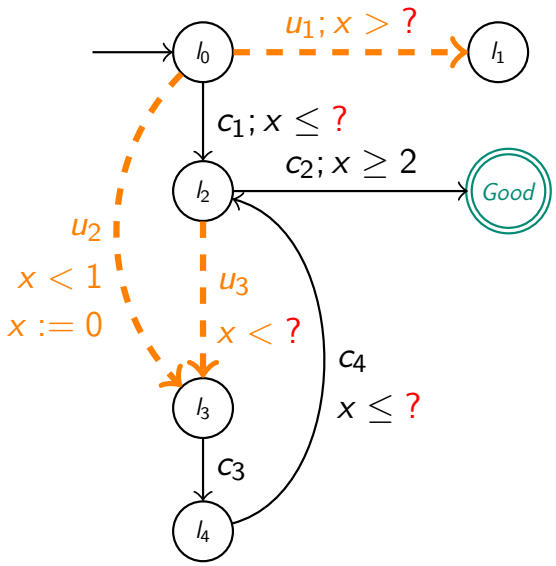

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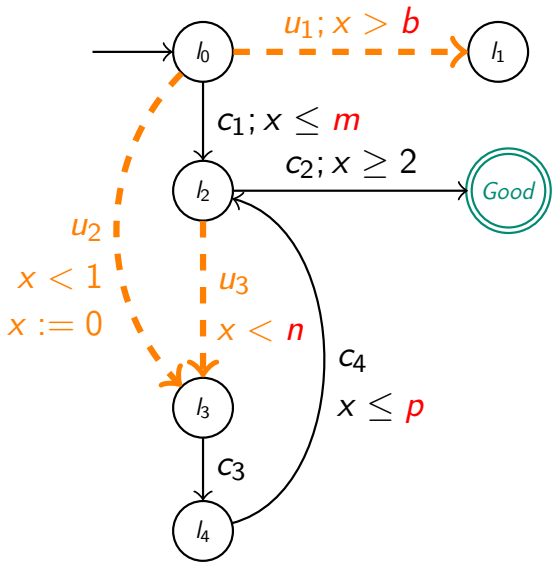

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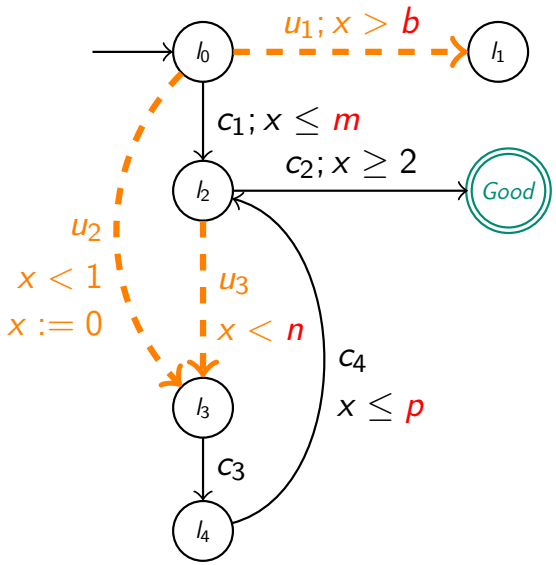

Reachability emptiness problem for PGA

Is there a parameter valuation v such that there is a winning strategy?

Decidability Issues for PGA

Theorem

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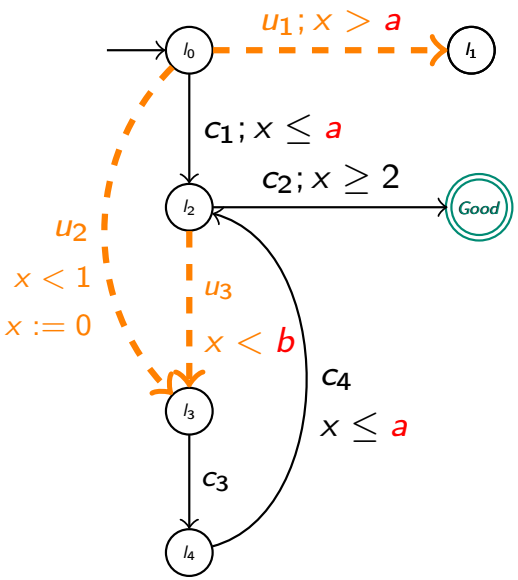
New subclass of PGA: L/U game automata



Syntactical restrictions:

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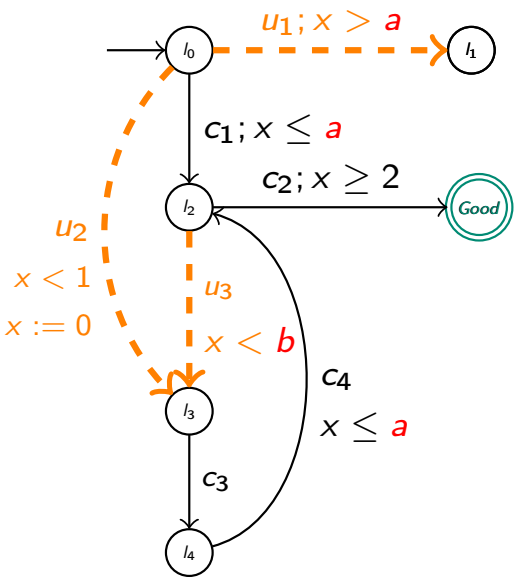

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Theorem

Reachability emptiness problem for L/U games is decidable.

Algorithm for Solving Parametric Timed Games

Symbolic on-the-fly algorithm for solving TGA [Cassez & al. '05]

- winning set of states

Extend the algorithm for the parametric approach

- set of parameter valuations with the set of winning states

We use extended notion of the symbolic state:

- parametric zone - set of pairs (w, v) satisfying clock constraint
- w - clock valuation
- v - parameter valuation

Theorem

In the case of termination, if the initial state belongs to the winning set of states, set of parameter valuations is obtained and the winning strategy can be extracted from the winning set of states.

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Conclusions

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Parametric model for timed games:

- Timed games with parameters
  - parametric timed games
- Reachability emptiness decidable subclass
  - L/U game automata
- Extension of the algorithm for solving timed games
  - parameter synthesis