Recent Results and Future Directions in Strategy Logic

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10th school about Modelling and Verifying Parallel Processes
Marseille, France, December 4th, 2012

Aim of our work

Idea

We are looking for a logic in which we can talk about the strategic behavior of agents in generic multi-player concurrent games.

Application

It can be used as a specification language for the formal verification and synthesis of modular and interactive systems.

Strategic reasoning

Example (Reactive synthesis)

Synthesize an interactive system that satisfies a given specification, independently of the possible sequences of inputs.

Nash equilibrium

Verify that all players of a game have optimal strategies (each player has a strategy such that it is rational for him to adhere to it assuming that all the other players also do so).

The previous logic ATL*

Alternating-time Temporal Logic [Alur, Henzinger and Kupferman, 2002]

$\langle\langle\{\alpha, \beta\}\rangle\rangle \mathsf{G} \neg \mathit{fail}$: "Agents α and β cooperate to ensure that a system (having possibly more than two processes (agents)) never enters a fail state".

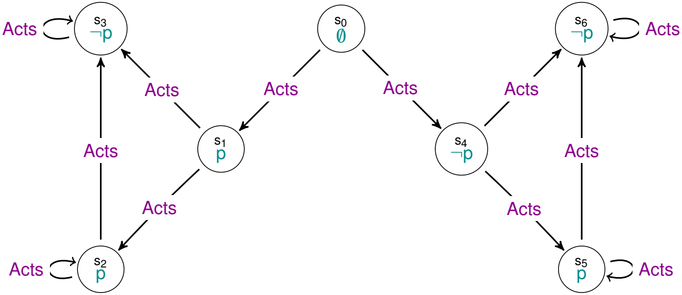
- Implicit strategies.
- 1 alternation of quantification.

Our contribution

We introduce *Strategy Logic* (SL) for explicit reasoning about strategies in multi-player concurrent games.

We also study a chain of more tractable syntactic fragments which results to be strictly more expressive than ATL*.

Underlying framework: the concurrent game structure



A Concurrent Game Structure is a graph in which each state is labeled by atomic propositions and each edge is labeled with the actions that agents can choose. A strategy maps histories of the game into actions. Plays are completely determined by the strategies.

Strategy Logic [Mogavero, Murano, and Vardi, 2010]

SL syntactically extends LTL by means of *strategy quantifiers*, the existential $\langle\langle x \rangle\rangle$ and the universal $[[x]]$, and *agent binding* (a, x).

Syntax

SL *formulas* are built inductively in the following way, where x is a variable and a an agent.

$$\varphi ::= \text{LTL} \mid \langle\langle x \rangle\rangle \varphi \mid [[x]] \varphi \mid (a, x) \varphi.$$

Informal semantics

- $\langle\langle x \rangle\rangle \varphi$: *"there exists a strategy x for which φ is true".*
- $[[x]] \varphi$: *"for all strategies x, it holds that φ is true".*
- $(a, x) \varphi$: *"φ holds, when the agent a uses the strategy x".*

Example: Failure is not an option

No failure property

"In a system $\mathcal{S}$ built on three processes, α, β, and γ, the first two have to cooperate in order to ensure that $\mathcal{S}$ never enters a failure state".

Three different formalization in SL.

- $\langle\langle x\rangle\rangle\langle\langle y\rangle\rangle[[z]](\alpha, x)(\beta, y)(\gamma, z)(G \neg \mathit{fail})$: α and β have two strategies, x and y, respectively, that, independently of what γ decides, ensure that a failure state is never reached.
- $\langle\langle x\rangle\rangle[[z]]\langle\langle y\rangle\rangle(\alpha, x)(\beta, y)(\gamma, z)(G \neg \mathit{fail})$: β can chose his strategy y dependently of that one chosen by γ.
- $\langle\langle x\rangle\rangle[[z]](\alpha, x)(\beta, x)(\gamma, z)(G \neg \mathit{fail})$: α and β have a common strategy x to ensure the required property.

Example: Multi-player Nash equilibrium

Nash equilibrium

Let $\mathcal{G}$ be a game with the n agents $\alpha_1, \ldots, \alpha_n$, each one having its own LTL goal $\psi_1, \ldots, \psi_n$. We want to know if $\mathcal{G}$ admits a Nash equilibrium, i.e., if there is a "best" strategy x_i w.r.t. the goal ψ_i, for each agent α_i, once all other strategies are fixed.

$$\varphi_{NE} \triangleq \langle\langle x_1 \rangle\rangle \cdots \langle\langle x_n \rangle\rangle (\alpha_1, x_1) \cdots (\alpha_n, x_n) (\bigwedge_{i=1}^{n} (\langle\langle y \rangle\rangle (\alpha_i, y) \psi_i) \rightarrow \psi_i).$$

Intuitively, if $\mathcal{G} \models \varphi_{NE}$ then $x_1, \ldots, x_n$ form a Nash equilibrium, since, when an agent α_i has a strategy y that allows the satisfaction of ψ_i, he can use x_i instead of y, assuming that the remaining agents $\alpha_1, \ldots, \alpha_{i-1}, \alpha_{i+1}, \ldots, \alpha_n$ use $x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n$.

Model-theoretic properties

	ATL*	...	...	...	SL
Bisimulation Invariance	Yes	...	...	...	No
State-tree	Yes	...	...	...	Yes
Bounded model property	Yes	...	...	...	No

Decision problems

	Model-checking	Satisfiability
SL	NONELEMENTARY-COMPLETE	Σ_1^1-HARD
...	...	...
...	...	...
...	...	...
ATL*	2EXPTIME-COMPLETE	2EXPTIME-COMPLETE

Decision problems

	Model-checking	Satisfiability
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... ...	... ...	... ...
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Why ATL* is more tractable than SL?

A possible solution

Answer

"SL allows to write formulas for which the satisfiability on a play depends, in some way, on the satisfiability on other independent plays."

Our results

- We introduce three fragments, called SL[NG], SL[BG], and SL[1G], with decreasing expressiveness. [Mogavero, Murano, Perelli, and Vardi, 2012]
- The less expressive fragment SL[1G] is anyway strictly more expressive than ATL* but still it does not have the ability to write such delicate formulas. The complexities of the decision problems for SL[1G] are the same as for ATL*.

Model-theoretic properties

	ATL*	SL[1G]	SL[BG]	SL[NG]	SL
Bisimulation Invariance	Yes	Yes	No	No	No
State-tree model	Yes	Yes	Yes	Yes	Yes
Bounded model property	Yes	Yes	No	No	No

Decision problems

	Model-checking	Satisfiability
SL	NONELEMENTARY-COMPLETE	Σ^1_1-HARD
SL[NG] SL[BG] SL[1G]	NONELEMENTARY-COMPLETE ? 2EXPTIME-COMPLETE	Σ^1_1-HARD Σ^1_1-HARD 2EXPTIME-COMPLETE
ATL*	2EXPTIME-COMPLETE	2EXPTIME-COMPLETE

Future works

Open problems

- SL[BG] model checking.
- SL on turn-based structures.

Possible extensions

- Graded Strategy Logic.
- Coalition Strategy Logic.
- Normative Systems with Strategy Logic.

Thank you very much for your attention!