Construction and Verification of Unfoldings for Petri Nets with Read Arcs

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Introduction

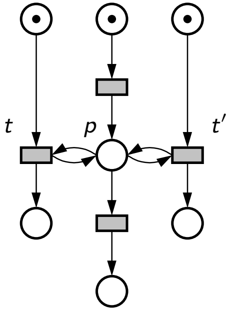
The problem

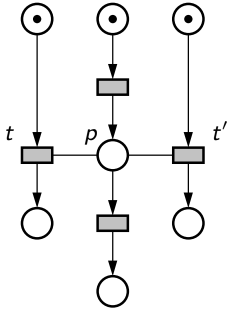
Verification of concurrent systems by means of the unfolding technique, when the system is modelled as a Petri net with read arcs.

- Unfolding up to exponentially more compact
- Unfolding algorithm more involved, but has better efficiency
- Reachability and deadlock-checking

Contextual Petri nets

- Contextual nets are Petri nets + read arcs
- Natural representation of notion *checking without consuming*





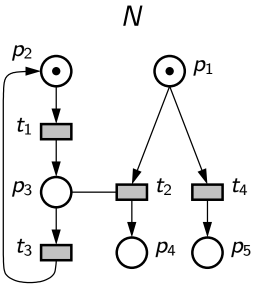
Notation

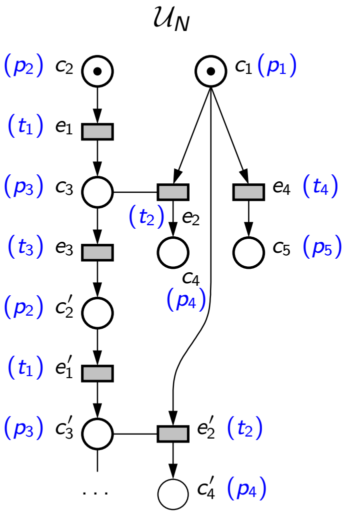
- A c-net is a tuple $\langle P, T, F, C, m_0 \rangle$
- ${}^\bullet x$ for preset, $x^\bullet$ for postset
- $\underline{t} = \{p \in P \mid (t, p) \in C\}$ for context

Example

$\underline{p} = \{t, t'\}$
$\underline{t} = \{p\}$

Contextual net unfoldings

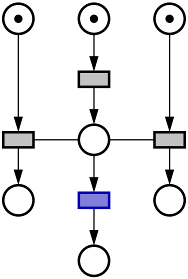




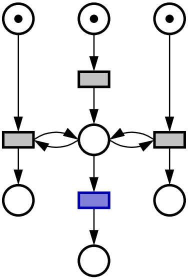
Remarks

- Labeling $f : \mathcal{U}_N \to N$
- $\mathcal{U}_N$ is marking-complete

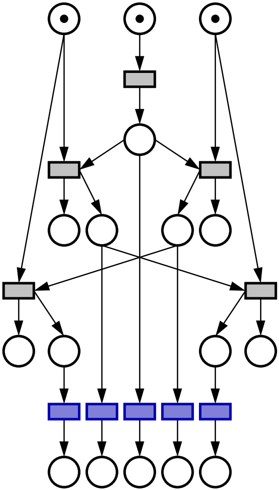
Contextual unfoldings exploit concurrent read access



A

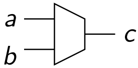


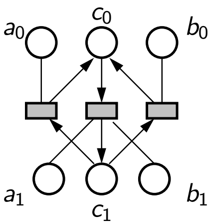
B

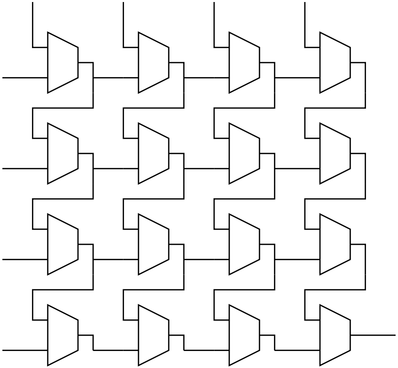


C

Asynchronous circuits







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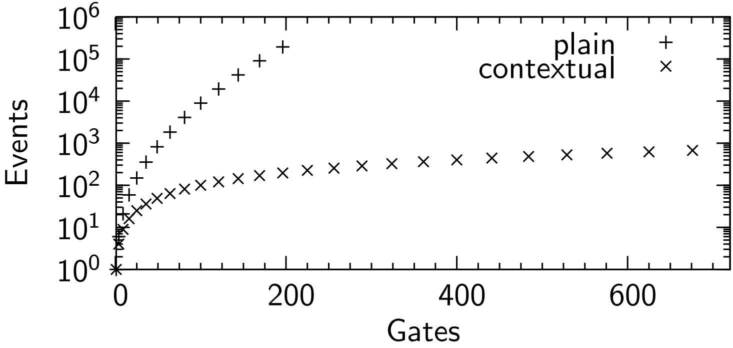

Computing prefix extensions

The problem

Given $\mathcal{P}_N$ and t, decide if we can extend $\mathcal{P}_N$ with e where $f(e) = t$

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Conditions c, c' are concurrent, $c \parallel c'$, iff some run marks them both

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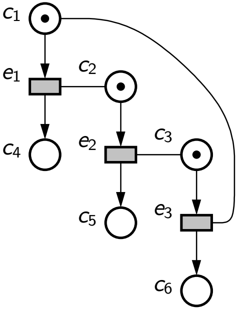
Proposition

Conditions $c_1, \ldots, c_n$ are coverable iff $c_i \parallel c_j$ holds for all $i, j \in \{1, \ldots, n\}$

However, for contextual unfolding...

...the same approach doesn't work:



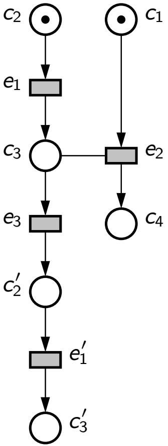
We have $c_4 \parallel c_5$, and $c_4 \parallel c_6$ and $c_5 \parallel c_6$ but $\{c_4, c_5, c_6\}$ is not coverable.

Histories for events and conditions

Definition

A history of e is a set of events H such that:

1. $e \in H$,
2. Events in H can be arranged to form a *run*,
3. *Any* run of the events of H fires e last.



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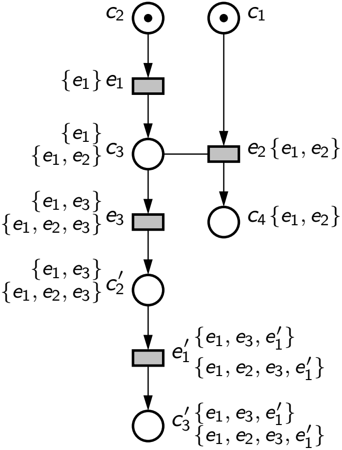

Remarks

- Enriched prefix: events and conditions annotated with histories
- A pair (c, H) is called enriched condition
- This is the working *data structure*

A concurrency relation for contextual nets

Definition

Two enriched conditions $\rho = (c, H)$ and $\rho' = (c', H')$ are concurrent, written $\rho \parallel \rho'$, iff:

$$\neg(H \# H') \quad \text{and} \quad c, c' \in (H \cup H')^\bullet$$

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Proposition

Conditions $c_1, \ldots, c_n$ coverable iff there exist histories $H_1, \ldots, H_n$ verifying

$$(c_i, H_i) \parallel (c_j, H_j) \text{ for all } i, j \in \{1, \ldots, n\}$$

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Proposition

Let $\rho = (c, H)$ and e be the last enriched condition and event appended to the prefix, let $\rho' = (c', H')$ be an arbitrary enriched condition. Then,

$$\rho \parallel \rho' \iff (c' \in e^{\bullet} \wedge H = H') \vee \left(c' \notin {}^{\bullet}e \;\wedge\; \bigwedge_{i=1}^{n} (\rho_i \parallel \rho') \;\wedge\; \underline{{}^{\bullet}e} \cap H' \subseteq H \right)$$

Experiments with Cunf

	Contextual		Ordinary		Ratios	
Net	Events	t_C	Events	t_P	t_C/t_P	t_C/t_R
bds_1.sync	1866	0.14	12900	0.51	0.27	0.54
byzagr4_1b	8044	2.90	14724	3.40	0.85	0.55
ftp_1.sync	50928	34.21	83889	76.74	0.45	0.30
furnace_4	95335	18.34	146606	40.39	0.45	0.42
key_4.fsa	4754	6.33	67954	2.21	2.86	1.47
rw_1w3r	14490	0.45	15401	0.38	1.18	0.65
q_1.sync	10722	1.13	10722	1.21	0.93	0.52
dpd_7.sync	10457	0.91	10457	0.88	1.03	0.92
elevator_4	16856	1.26	16856	2.01	0.63	$>$0.01
rw_12.sync	98361	3.10	98361	3.95	0.78	0.41
rw_2w1r	9241	0.40	9241	0.30	1.33	0.04

- Contextual unfolding smaller or equal than ordinary unfolding
- And in general faster than unfolding the plain encoding

Encoding deadlock and reachability into SAT

From a marking-complete unfolding prefix $\mathcal{P}$, we construct

- $\phi_{\mathcal{P}}^{\text{dead}}$, satisfiable iff N contains a deadlock
- $\phi_{\mathcal{P}}^{\text{reach, M}}$, satisfiable iff places M are coverable in N

Encoding deadlock and reachability into SAT

From a marking-complete unfolding prefix $\mathcal{P}$, we construct

- $\phi_{\mathcal{P}}^{\text{dead}}$, satisfiable iff N contains a deadlock
- $\phi_{\mathcal{P}}^{\text{reach, M}}$, satisfiable iff places M are coverable in N

Both formulas characterize configurations and reachable markings:

$$\begin{aligned} \phi_{\mathcal{P}}^{\text{dead}} &:= \phi_{\mathcal{P}}^{\text{conf}} \wedge \phi_{\mathcal{P}}^{\text{disable}} \\ \phi_{\mathcal{P}}^{\text{reach, M}} &:= \phi_{\mathcal{P}}^{\text{conf}} \wedge \phi_{\mathcal{P}}^{\text{mark, M}} \end{aligned}$$

where $\phi_{\mathcal{P}}^{\text{conf}}$ is defined as

$$\phi_{\mathcal{P}}^{\text{causal}} \wedge \phi_{\mathcal{P}}^{\text{sym}} \wedge \phi_{\mathcal{P}}^{\text{asym}}$$

- Implementation runs twice faster than the best tool we found

Summary

- Contextual unfoldings are up to exponentially more compact
- In our benchmark, verification based on contextual unfoldings performs better than existing methods
- Unfolder and unfolding-based analysis tool available at:

  `www.lsv.ens-cachan.fr/~rodriguez/tools/cunf/`

Current and future work

- Contextual merged processes
- Application in diagnosis
- We are searching for concurrent systems to evaluate our algorithms !!

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Thank you for your attention

References

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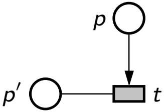
Computing possible extensions

Net

Unfolding



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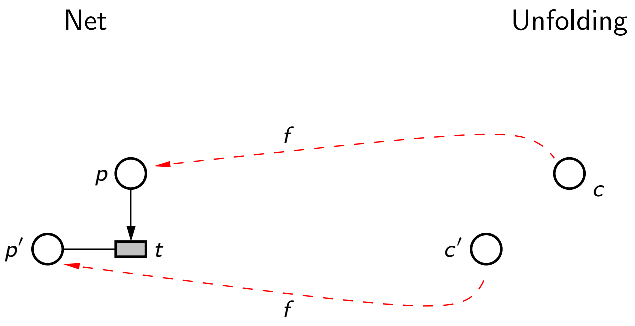

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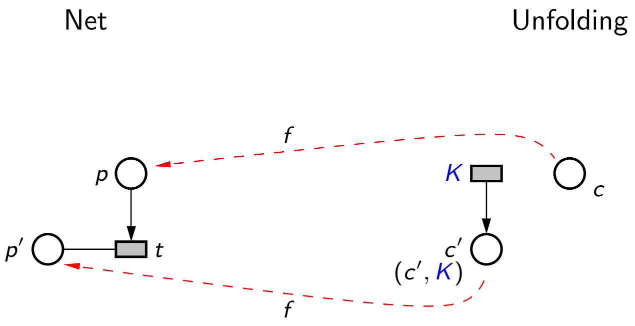

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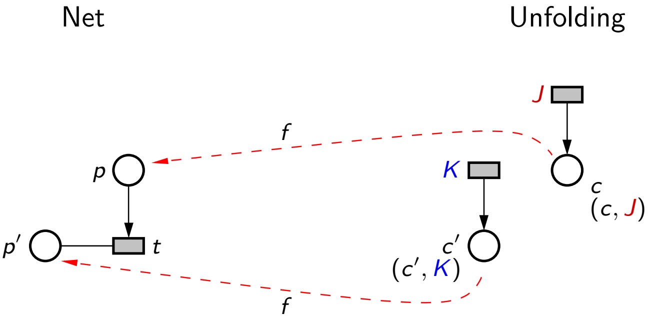

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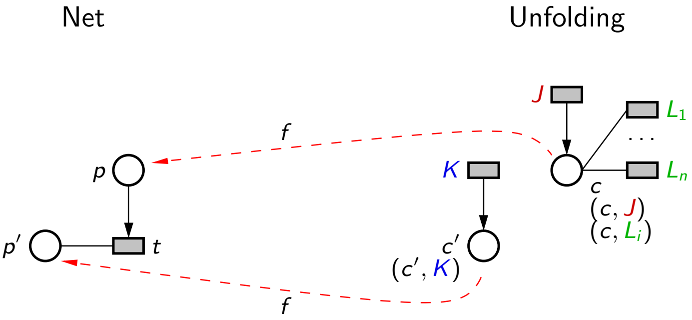

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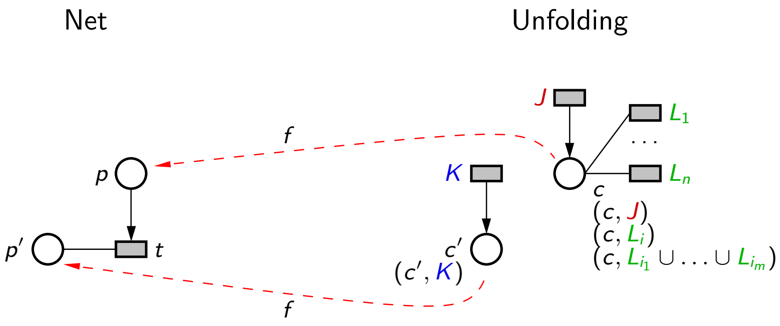

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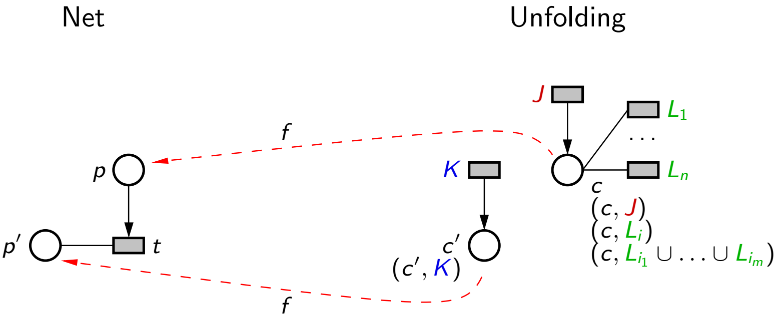

$(c, J) \parallel (c', K)$

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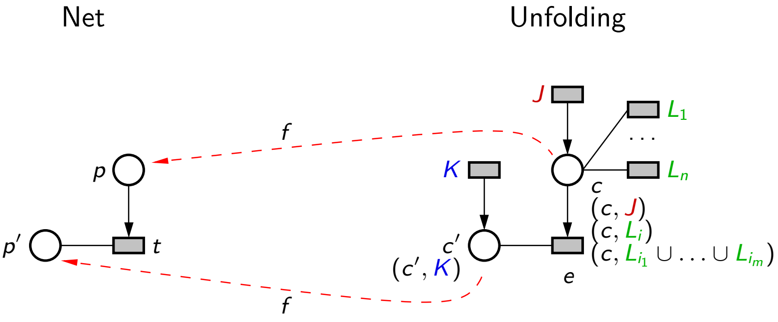

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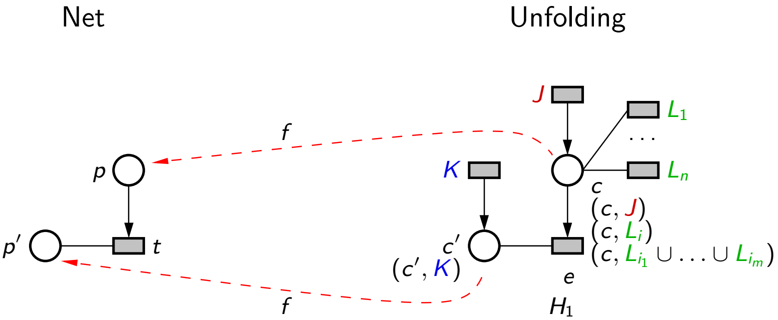

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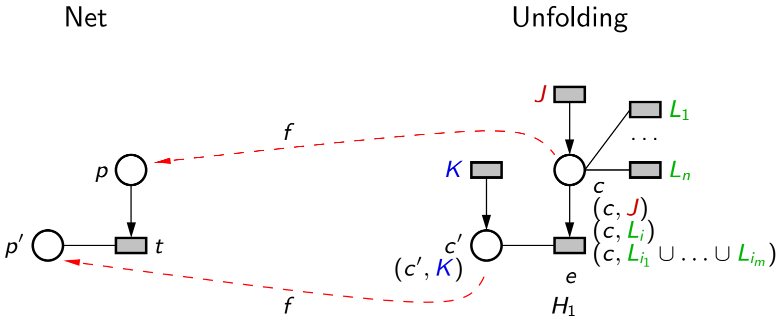

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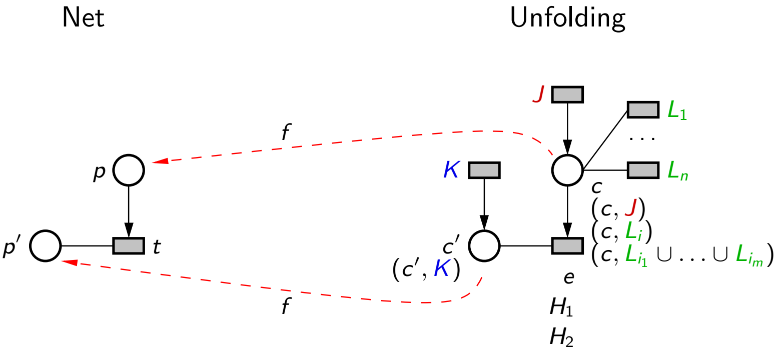

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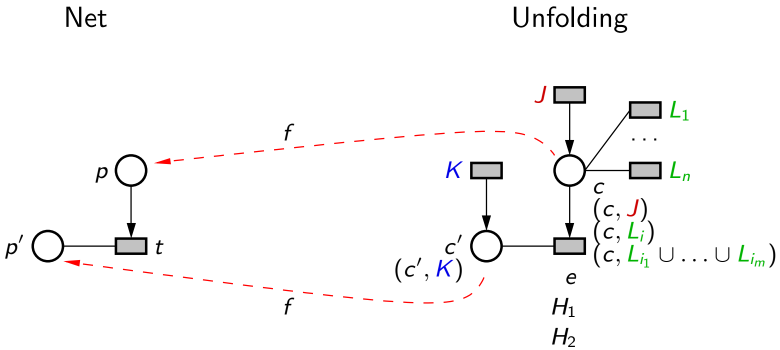

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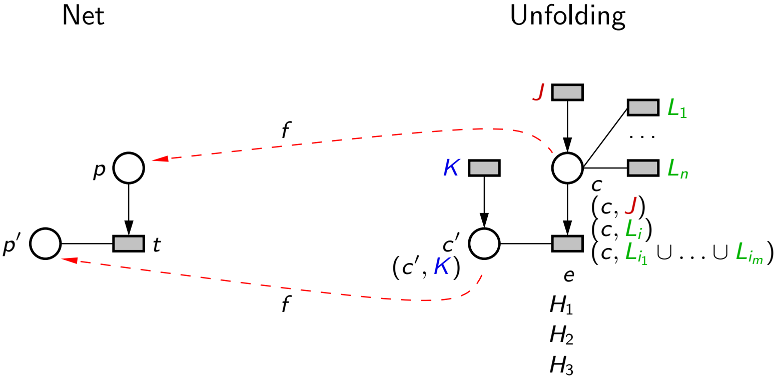

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(c, L_i) \parallel (c', K) &\quad \rightsquigarrow H_2 = L_i \cup K \\
(c, L_{i_1} \cup \ldots \cup L_{i_m}) \parallel (c', K) &\quad \rightsquigarrow H_3 = L_{i_1} \cup \ldots \cup L_{i_m} \cup K
\end{aligned}
$$

Contextual unfolding — inductive definition

For a 1-safe contextual net $N = \langle P, T, F, C, m_0 \rangle$, the full unfolding $\mathcal{U}_N = \langle P', T', F', C', m'_0 \rangle$ is the 1-safe acyclic contextual net defined by the next inductive rules:

Mapping $f : \mathcal{U}_N \to N$ labels every event $\langle A, B, t \rangle$ with t and every condition $\langle e, p \rangle$ with p.