On Minimality and Equivalence of Petri Nets

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Outline

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Petri nets: The networks

The networks

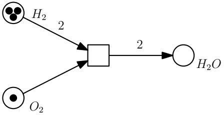
are graphs $G = (P, T, A, w)$ with

- P set of involved **components** ("places" $\bigcirc$),
- T set of involved **reactions** ("transitions" $\square$),
- interconnected by directed **links** in A ("arcs" $\rightarrow$).

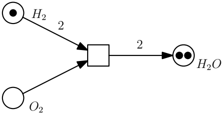
Each place $p \in P$ can be marked with an integral number x_p of tokens.
A **state** can be represented as a vector $x \in \mathbb{N}^{|P|}$ with entries x_p for all $p \in P$.

A transition $t \in T$ is **enabled** at a state x if there are enough tokens available on the pre-places of t.
Switching t transfers x into a new system state, denoted by $x \rightarrow x'$.

$x = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$



$\Rightarrow$



$\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = x'$

Deterministic extended Petri nets

Deterministic extended Petri nets

A triple $(\mathcal{P}, \mathrm{cap}, \mathcal{O})$

- extended Petri net $\mathcal{P} = (P, T, A_S \cup A_C, w)$
- capacity on places $\mathrm{cap} : P \to \mathbb{N}$
- set of priorities $\mathcal{O}$ (e.g., partial order)

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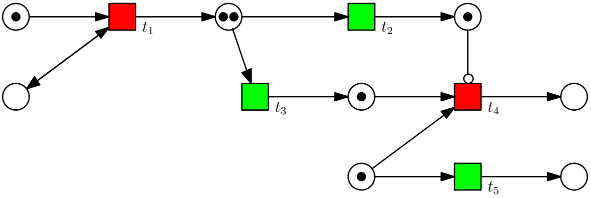

$\mathcal{O} = \{t_1 < t_3, t_2 < t_3, t_5 < t_3\}$

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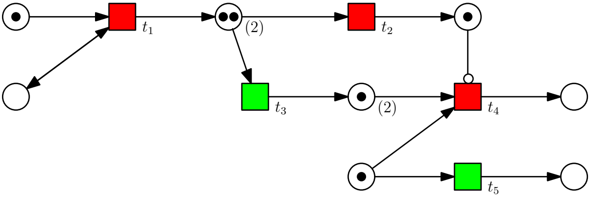

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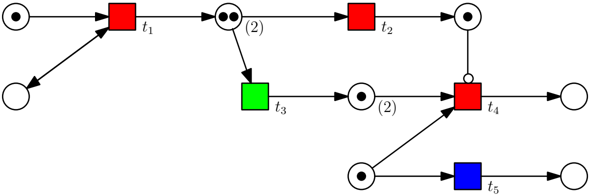

$$\mathcal{O} = \{t_1 < t_3, t_2 < t_3, t_5 < t_3\}$$

Network reconstruction algorithm

Input:

- a set P of components (considered to be crucial for the studied phenomenon)
- a capacity cap for each component
- experimental time-series data $\mathcal{X}' = (x^1, \ldots, x^m)$ obtained by stimulating a biological system and observing how its states change over time

Output:

all deterministic extended Petri nets explaining $\mathcal{X}'$ (so called $\mathcal{X}'$-deterministic extended Petri nets):

- all generated nets have the same set of places P and capacity cap
- there are enough transitions to simulate all observed state changes
- A_C and $\mathcal{O}$ are so that the experiments can exactly be reproduced

Output: Minimal solution sets

To keep the solution set small while guaranteeing its completeness, return only "minimal" $\mathcal{X}'$-deterministic extended Petri nets.

How to obtain "minimal" solutions?

Easy in standard Petri nets:

- remove unnecessary transitions.

Difficult in deterministic extended Petri nets: we can also

- remove unnecessary priorities and/or control-arcs,
- replace priorities by control-arcs (or vice versa).

Idea:

- consider **equivalence classes** of deterministic extended Petri nets having the same places and transitions,
- select **representatives** being minimal w.r.t. priorities and control-arcs.

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Equivalence and Inclusion

$\mathcal{X}'$-equivalence

Two $\mathcal{X}'$-deterministic extended Petri nets $(\mathcal{P}, \text{cap}, \mathcal{O})$, $(\hat{\mathcal{P}}, \text{cap}, \hat{\mathcal{O}})$ are **$\mathcal{X}'$-equivalent** if they only differ in the set of priorities and/or control-arcs.

Consider an $\mathcal{X}'$-deterministic extended Petri nets $(\mathcal{P}, \text{cap}, \mathcal{O})$. A sequence of transitions $t^1, \ldots, t^k$ is an **$\mathcal{O}$-feasible switching sequence** for x^1 in $\mathcal{P}$ if for all j

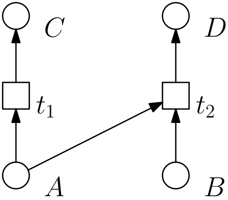
- x^{j+1} is the successor state of x^j switching t^j,
- there does not exist a $t \in T$ enabled in x^j with $(t^j < t) \in \mathcal{O}$.

Inclusion relation

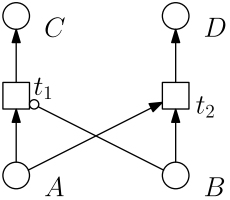
Consider two $\mathcal{X}'$-equivalent extended Petri nets $(\mathcal{P}, \text{cap}, \mathcal{O})$ and $(\hat{\mathcal{P}}, \text{cap}, \hat{\mathcal{O}})$. We say $\mathcal{P}$ is **included** in $\hat{\mathcal{P}}$, denoted by $\mathcal{P} \subseteq \hat{\mathcal{P}}$, if and only if for all states $x \in \mathcal{X}$, every $\hat{\mathcal{O}}$-feasible switching sequence for x in $\hat{\mathcal{P}}$ is an $\mathcal{O}$-feasible switching sequence for x in $\mathcal{P}$.

Inclusion relation: Example

$\mathcal{P} =$

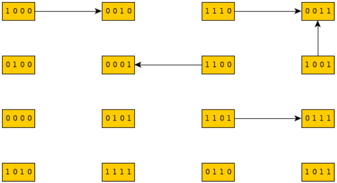


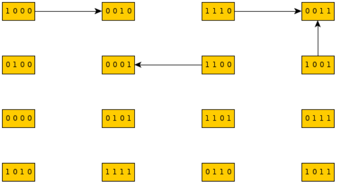
$\hat{\mathcal{P}} =$



$$\mathcal{O} = \{t_1 < t_2\}$$

$$\hat{\mathcal{O}} = \emptyset$$





$\Rightarrow \mathcal{P} \subset \hat{\mathcal{P}}$

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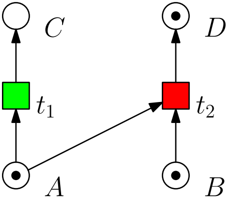

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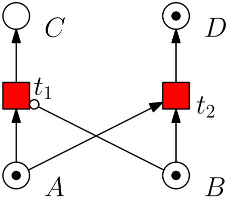

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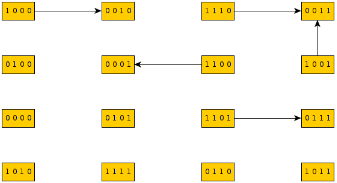

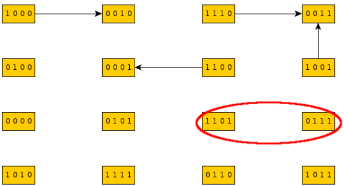

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Minimal $\mathcal{X}'$-deterministic extended Petri net

Among all $\mathcal{X}'$-equivalent extended Petri nets, $(\mathcal{P}, \text{cap}, \mathcal{O})$ is **minimal** if and only if $(\mathcal{P}, \text{cap}, \mathcal{O})$ does neither have unnecessary elements nor another $\mathcal{X}'$-deterministic extended Petri net $(\hat{\mathcal{P}}, \text{cap}, \hat{\mathcal{O}})$ being $\mathcal{X}'$-equivalent to $(\mathcal{P}, \text{cap}, \mathcal{O})$ is included in $\mathcal{P}$.

In order to classify $\mathcal{X}'$-equivalent extended Petri nets for inclusion we consider $(\mathcal{P}, \text{cap}, \mathcal{O})$ and $(\hat{\mathcal{P}}, \text{cap}, \hat{\mathcal{O}})$ and distinguish the following four cases:

1. $\mathcal{O} \subset \hat{\mathcal{O}}$ and $A_C = \hat{A}_C$,
2. $\mathcal{O} = \hat{\mathcal{O}}$ and $A_C \subset \hat{A}_C$,
3. $\mathcal{O} \subset \hat{\mathcal{O}}$ and $A_C \subset \hat{A}_C$,
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Classification of $\mathcal{X}'$-Deterministic Extended Petri Nets

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$A_C \subset \hat{A}_C$			
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Theorem (Case 1)

Let $(\mathcal{P}, \text{cap}, \mathcal{O})$ and $(\hat{\mathcal{P}}, \text{cap}, \hat{\mathcal{O}})$ be two $\mathcal{X}'$-equivalent extended Petri nets with $\mathcal{O} \subset \hat{\mathcal{O}}$ and $A_C = \hat{A}_C$. Then $\mathcal{P} \subseteq \hat{\mathcal{P}}$ holds.

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$A_C \subset \hat{A}_C$	?	?	
$A_C = \hat{A}_C$	$\mathcal{P} \subseteq \hat{\mathcal{P}}$	-	$\hat{\mathcal{P}} \subseteq \mathcal{P}$
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Remark (Cases 2 and 3)

Let $(\mathcal{P}, \text{cap}, \mathcal{O})$ and $(\hat{\mathcal{P}}, \text{cap}, \hat{\mathcal{O}})$ be two $\mathcal{X}'$-equivalent extended Petri nets with $\mathcal{O} \subseteq \hat{\mathcal{O}}$ and $A_C \subset \hat{A}_C$. Then, in general, neither $\mathcal{P} \subseteq \hat{\mathcal{P}}$ nor $\hat{\mathcal{P}} \subseteq \mathcal{P}$ follows.

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Conjecture (Case 4)

Let $(\mathcal{P}, \text{cap}, \mathcal{O})$ and $(\hat{\mathcal{P}}, \text{cap}, \hat{\mathcal{O}})$ be two $\mathcal{X}'$-equivalent Petri nets with $\hat{\mathcal{O}} \subset \mathcal{O}$ and $A_C \subset \hat{A}_C$. Furthermore, let every control-arc in $\hat{A}_C$ be necessary.
If for all $(t < t') \in \mathcal{O} \setminus \hat{\mathcal{O}}$ the following properties hold:

- there exists a "specific" control-arc $(p, t) \in \hat{A}_C \setminus A_C$,
- $(t < t')$ is strictly necessary in $\mathcal{O}$,
- there does not exist a transition t'' with $(t'' < t) \in \hat{\mathcal{O}}$,
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- control-arcs,
- capacities on places,
- priorities on transitions.

We defined an **inclusion relation** on sets of $\mathcal{X}'$-equivalent extended Petri nets, as well as the notion of a **minimal** representative of this set.

We examined $\mathcal{X}'$-equivalent extended Petri nets $(\mathcal{P}, \text{cap}, \mathcal{O})$ and $(\hat{\mathcal{P}}, \text{cap}, \hat{\mathcal{O}})$:

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$A_C = \hat{A}_C$	$\mathcal{P} \subseteq \hat{\mathcal{P}}$	-	$\hat{\mathcal{P}} \subseteq \mathcal{P}$
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Future work: prove the conjecture, fill gaps in table and integrate results in reconstruction algorithm.

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$A_C = \hat{A}_C$	$\mathcal{P} \subseteq \hat{\mathcal{P}}$	-	$\hat{\mathcal{P}} \subseteq \mathcal{P}$
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$A_C \subset \hat{A}_C$	?	?	"$\mathcal{P} \subseteq \hat{\mathcal{P}}$"
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$\hat{A}_C \subset A_C$	"$\hat{\mathcal{P}} \subseteq \mathcal{P}$"	?	?

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Thank You

Case 2) & 3) $\mathcal{O} \subseteq \hat{\mathcal{O}}$ and $A_C \subset \hat{A}_C$

Remark

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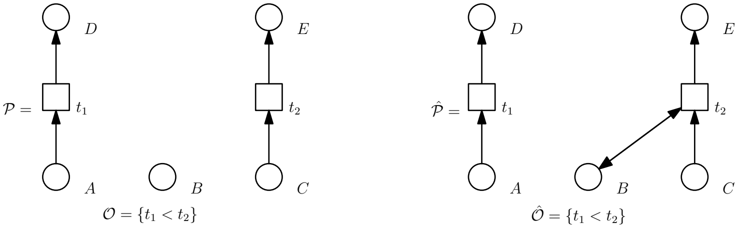

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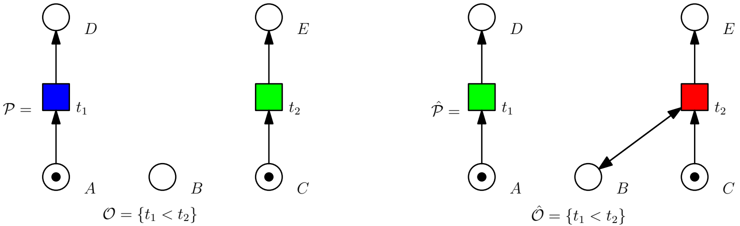
Let $(\mathcal{P}, \text{cap}, \mathcal{O})$ and $(\hat{\mathcal{P}}, \text{cap}, \hat{\mathcal{O}})$ be two $\mathcal{X}'$-equivalent extended Petri nets with $\mathcal{O} \subseteq \hat{\mathcal{O}}$ and $A_C \subset \hat{A}_C$. Then, in general, neither $\mathcal{P} \subseteq \hat{\mathcal{P}}$ nor $\hat{\mathcal{P}} \subseteq \mathcal{P}$ follows.



Input: Experimental time-series data

We consider **experimental time-series data**, where

- set of **observed states** $\mathcal{X}'$
- **stimulations** to the network (dashed arcs)
- observed **responses** (solid arcs)